

Name _____

Section _____

Instructor _____

Instructions: There 13 problems on 10 pages. Please check that your copy of the exam has all the pages and all the problems numbered 1 to 13.

Work in a neat and organized manner. **Show ALL work** on all problems, and simplify answers unless otherwise specified. Full credit will not be given unless your work is clearly shown. On definite integrals **no credit** will be given for numerical approximations to the answer without all the supporting work.

A scientific calculator will be permitted on the final exam; however, calculators with graphic, word-processing, symbolic manipulation or programming capabilities will **not** be permitted for this exam.

Problem	Possible	Score
1	28	
2	10	
3	16	
4	12	
5	12	
6	20	
7	21	
8	16	
9	12	
10	15	
11	12	
12	12	
13	14	
Total	200	

(28) 1. Compute the derivative of the following functions. **Do not** simplify.

(a) $f(x) = -3x^4 + 6\sqrt{x} + 10$

(b) $f(x) = (x^2 - 2x) \ln(x^2 + 1)$

(c) $f(x) = \frac{e^x + 1}{e^x - 1}$

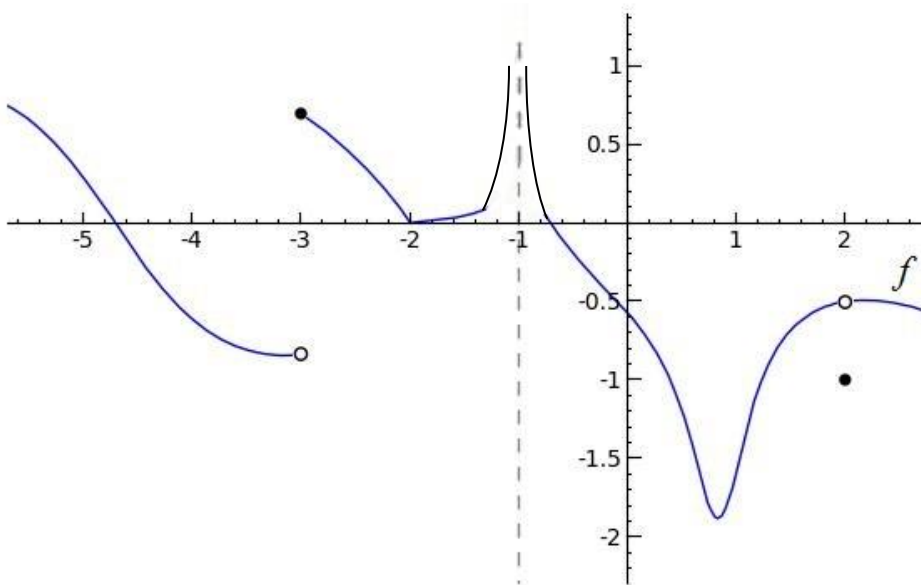
(d) $f(x) = \sqrt{10 + x^3}$

(10) 2. Calculate the following limits.

(a) $\lim_{x \rightarrow -1} \frac{x^2 + 3x - 4}{x^2 - x}$

(b) $\lim_{x \rightarrow \infty} \frac{1 + x - x^3}{8x^4 - 5}$

(16) 3. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



****There is a vertical asymptote at $x = -1$.**

(a) $\lim_{x \rightarrow -3^+} f(x) =$

(c) $\lim_{x \rightarrow 2} f(x) =$

(b) $\lim_{x \rightarrow -3} f(x) =$

(d) Is f continuous at $x = -1$?

(12) 4. Find the equation of the tangent line to the graph of the function

$f(x) = \frac{3}{x-1}$ at the point (2,3). State your answer in slope-intercept form.

(12) 5. Ultra Mobile has costs that are given by

$$C(x) = 2000 + 90x + 0.2x^2$$

and price given by

$$p(x) = 210 - 0.3x$$

where x is the number of SIM cards produced. What is the number of SIM cards that Ultra Mobile must produce and sell in order to maximize profit? (Recall that the revenue is $R(x)=px$.) You must prove this using a derivative test.

(20) 6. Given the function $f(x) = x^3 - 3x^2 - 9x + 5$. Use calculus to find:

a) the critical value(s)

b) Find where $f(x)$ is increasing and decreasing. Use interval notation.

c) Determine the relative extrema. Classify as max or min. Use (x,y) form.

d) Find where $f(x)$ is concave up and concave down. Use interval notation.

e) Find inflection points in (x,y) form.

(21) 7. Compute the following integrals.

(a) $\int \left(4x^{1/2} + \frac{3}{x^2} - 12x \right) dx$

(b) $\int 5xe^{-3x^2} dx$

(c) $\int_2^5 \frac{2x}{\sqrt{x^2+12}} dx$

(16) 8. Given the two functions:

$$f(x) = x^2 - 3x \text{ and } g(x) = x$$

(a) Find the ordered pairs where f and g intersect.

(b) Find the area bounded by the graphs of f and g . (*Hint*: Draw a sketch first.)

- (12) 9. According to the U.S. Census Bureau, the population of the United States can be approximated by

$$P(t) = 282.3e^{0.01t}$$

where P is in millions and t is the number of years since 2000.

Find the average value of the population from 2002 to 2006 (i.e. from $t = 2$ to $t = 6$).

- (15) 10. Let

$$f(x, y) = -x^2y - 4x^4 + \frac{x}{y} . \text{ Find:}$$

(a) f_y

(b) f_{yx}

(c) $f_{yx}(1, 2)$

(12) 11. Find where the tangent line is horizontal: $f(x) = x^3 - 2x^2 - 4x + 4$. List in (x,y) form.

(12) 12. Let

$$f(x, y) = x^3 - 3xy + y^3.$$

The critical points of $f(x, y)$ are $(0, 0)$, $(1, 1)$. Identify each critical point as a relative minimum, a relative maximum, or a saddle point, showing work using the D -test.

- (14) 13. Suppose x TV's are produced at one factory, and y TV's are produced at a second factory.

Use the method of Lagrange multipliers to find the minimum value of the company's cost function

$$C(x, y) = 6x^2 + 12y^2$$

subject to the constraint that 90 TV's are produced total, i.e. $x + y = 90$.