Fall 2025 – SOLUTIONS

(10 pts) 1. Calculate the following limits.

(a) $\lim_{x\to -1}(x^3+1)^2(x^4+5)$ $= ((-1)^3+1)^2((-1)^4+5)$

$= (-1+1)^2(1+5)$

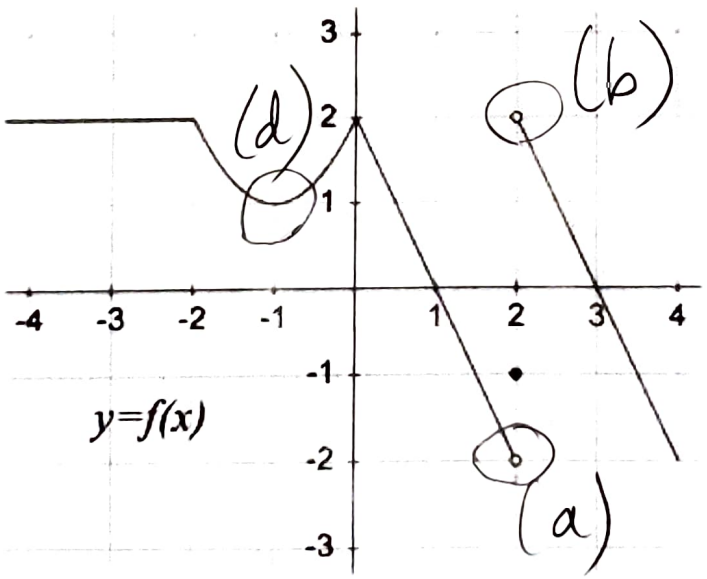
$= 0^2(6) = 0$

(b) $\lim_{x\to 3}\frac{x^2-8x+15}{x^2+x-12}$ $= \lim_{x\to 3}\frac{(x-5)(x-3)}{(x+4)(x-3)} = \lim_{x\to 3}\frac{x-5}{x+4}$

$= \frac{3-5}{3+4}$

$= \frac{-2}{7}$

(12 pts) 2. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a) $\lim_{x\to 2^-} f(x) = -2$

(b) $\lim_{x\to 2^+} f(x) = 2$

(c) $\lim_{x\to 2} f(x) =$ DNE

LHL ≠ RHL

(d) $\lim_{x\to -1} f(x) = 1$

(28 pts) 3. Compute the derivative of the following functions. **Do NOT** simplify.

(a) $f(x)=\frac{8\sqrt{x}}{x+7} = \frac{8x^{1/2}}{x+7}$ quotient rule

$$f'(x)= \frac{(x+7)(4x^{-1/2})-(8x^{1/2})(1)}{(x+7)^2}$$

(b) $f(x)=\ln(x^4+x-6)+e^{-x}$

$$f'(x)= \frac{4x^3+1}{x^4+x-6} - e^{-x}$$

(c) $g(x)=(2e^{x^2}-x)\left(\frac{4}{3}x^2+6x-10\right)$ ~~product~~ product rule

$$g'(x)=(2e^{x^2}-x)\left(\frac{8}{3}x+6\right)+\left(\frac{4}{3}x^2+6x-10\right)\cdot(2\cdot 2xe^{x^2}-1)$$

(d) $h(x)=(x+2)^{1/3}+2\ln x$

$$h'(x)=\frac{1}{3}(x+2)^{-2/3}(1)+2\frac{1}{x}$$

(12 pts) 4. A hotel owner notices that she rents y rooms per night when the price is x dollars per room, with $y=200-2x$.

a. Find $R(x)$, the total revenue generated per night when the price of each room is x dollars.

$$R(x)=x(200-2x) = -2x^2+200x$$

b. Find the relative extremum of R, and interpret this result.

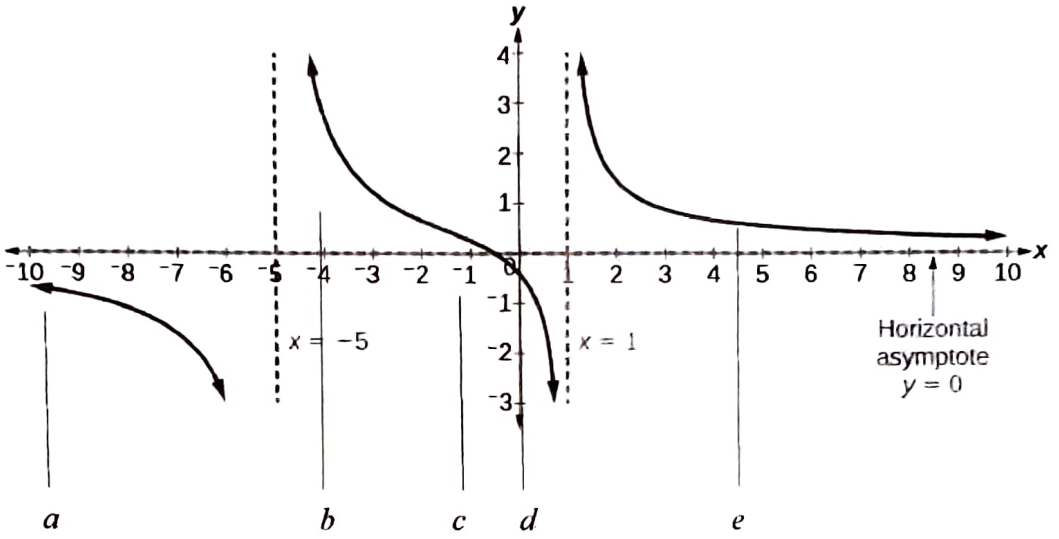
$$R'(x)=-4x+200=0$$
$$-4x=-200$$
$$x=50$$

$$R''(x)=-4$$
$$R''(50)=-4<0$$
$\Rightarrow$ max.

$$R(50)=-5000+10{,}000$$
$$=5000$$

She brings in a maximum revenue of \$5000 when she rents 50 rooms per night.

(12 pts) 5. Referring to the given graph, list the labeled value(s) of x, if applicable, at which the derivative has the stated property.



a) $f'(x)$ is positive — never
incr

b) $f''(x)$ is positive b, e
Conc. ↑

c) $f'(x)$ is negative a, b, c, d, e
decr

d) $f''(x)$ is negative a, c, d
Conc. ↓

(8 pts) 6. Marisa just had her 12th birthday. Her grandmother sets up a trust fund that, when Marisa turns 21, will be worth $50,000. Assuming that interest is compounded continuously at an annual rate of 3.75%, what is the present value of this trust fund?

$$PV = 50{,}000\, e^{-0.0375(9)}$$
$$= 50{,}000\, e^{-0.3375}$$
$$\approx 50{,}000\,(0.71355197)$$
$$\approx \$35{,}677.60$$

(12 pts) 7. a. Find the marginal cost at a production level of 1000 units for the cost function.

$C(x) = 2\sqrt{x} + \frac{x^2}{8000}$. $= 2x^{1/2} + \frac{1}{8000}x^2$

$C'(x) = x^{-1/2} + \frac{1}{4000}x$

$C'(1000) = \frac{1}{\sqrt{1000}} + \frac{1}{4} \approx \$0.28$

b. Explain what your answer means.

The cost of the 1001th unit will be approximately $0.28.

(16 pts) 8. Let $f(x,y) = 3x^2y^3 - e^{x^4} + x$. Find:

(a) $f_x = 6xy^3 - 4x^3e^{x^4} + 1$

(b) $f_{xx} = 6y^3 - [4x^3 \cdot 4x^3e^{x^4} + e^{x^4} \cdot 12x^2]$

(c) $f_y(4,1) = 3x^2 \cdot 3y^2 = 9x^2y^2$

at (4,1): $9(16)(1) = 144$

(12 pts) 9. Find the producers' surplus for wedding cakes at a bakery at the equilibrium price for the given supply and demand functions:

$$D(x) = -\frac{4}{15}x + 15\,,\ S(x) = \frac{1}{5}x + 1$$

$$-\frac{4}{15}x + 15 = \frac{1}{15}x + 1$$

$$14 = -\frac{5}{5}x$$

$$x = 42 \qquad S(x) = 3.8$$

$$PS = 42 \cdot 3.8 - \int_0^{42} \frac{1}{15}x + 1\, dx$$

$$= 159.6 - \left[\frac{1}{30}x^2 + x\right]_0^{42}$$

$$= 159.6 - [58.8 + 42 - 0]$$

$$= \$58.80$$

(12 pts) 10. For 12 years, Janice deposits $1500 every 3 months into a retirement account that has an annual interest rate of 3.25%, compounded quarterly. Find the future value of Janice's annuity.

$$FV = 1500\left[\frac{\left(1 + \frac{0.0325}{4}\right)^{4 \cdot 12} - 1}{\frac{0.0325}{4}}\right] \qquad \frac{0.0325}{4} = 0.008125$$

$$\approx \$87{,}628.64$$

b. What was Janice's personal contribution to the account?

$4 \cdot 12 = 48$ pymts of \$1500 each

$(48)(1500) = \$72{,}000$

(14 pts) 11. Compute the following indefinite integrals.

(a) $\int \left(12x^{1/4} - \frac{3}{x} + 50\right) dx = 12 \cdot \frac{4}{5} x^{5/4} - 3\ln x + 50x + C$

$= \frac{48}{5} x^{5/4} - 3\ln x + 50x + C$

(b) $\int (x^3 + 7)^4 \, 3x^2 \, dx$ $\quad u = x^3 + 7$

$du = 3x^2 dx$

$\int u^4 du = \frac{1}{5} u^5 + C$

$= \frac{1}{5}(x^3 + 7)^5 + C$

(14 pts) 12. Suppose a function is given by

$$P(x, y) = x^4 + y^4 - 4xy + 1.$$

The critical points of $P(x, y)$ are (0, 0), (1, 1), and (-1, -1). Identify each critical point as a relative minimum, a relative maximum, or a saddle point.

$P_x = 4x^3 - 4y$ $\quad P_y = 4y^3 - 4x$

$P_{xx} = 12x^2$ $\quad P_{yy} = 12y^2$

$P_{xy} = -4$

$(0,0)$: $D = 0 \cdot 0 - (-4)^2 = -16 < 0 \Rightarrow$ saddle ~~point~~

$(1,1)$: $D = 12 \cdot 12 - (-4)^2 = 144 - 16 = 128 > 0$

$P_{xx}(1,1) = 12(1)^2 = 12 > 0$

$\Rightarrow$ rel. min.

$(-1,-1)$: $D = 12 \cdot 12 - (-4)^2 = 128 > 0$

$P_{xx}(-1,-1) = 12(-1)^2 = 12 > 0$

$\Rightarrow$ rel. min.

(10 pts) 13. Solid Rock Industries determines that the marginal cost, $C'(x)$, of producing the xth climbing harness is given by $C'(x) = x^3 - x$. Find the total-cost function, C, assuming that $C(x)$ is in dollars and that fixed costs are \$6500. $\longrightarrow (0, 6500)$

$$C(x) = \int x^3 - x \, dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 + C$$

$$6500 = 0 - 0 + C$$
$$C = 6500$$

$$C(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 6500$$

(16 pts) 14. Using the two curves $f(x) = 2x - x^2$ and $g(x) = 2x^2 - 4x$:

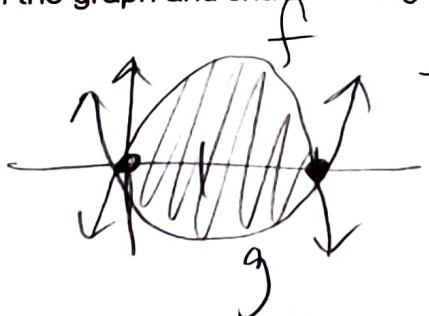
a) Find the points of intersection as ordered pairs.

$$2x - x^2 = 2x^2 - 4x$$
$$3x^2 - 6x = 0$$
$$3x(x-2) = 0$$
$$x = 0, 2$$

$f(0) = 0 \quad (0,0)$

$f(2) = 0 \quad (2,0)$

b) Sketch the graph and shade the region.



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c) Find the area bounded by the two curves.

$$\text{Area} = \int_0^2 2x - x^2 - (2x^2 - 4x)\, dx$$

$$= \int_0^2 -3x^2 + 6x = 0 = -x^3 + 3x^2 \Big]_0^2$$

$$= -8 + 12 - 0 = 4$$

(12 pts) 15. Using the Method of Lagrange Multipliers, minimize
$f(x,y) = 2y^2 - 3x^2$ subject to the constraint $g(x,y) = 4x + 4y - 16$.
Give where the minimum occurs as an ordered pair, and then state the minimum value.

$$F(x,y,\lambda) = 2y^2 - 3x^2 - \lambda(4x+4y-16)$$

$$F_x = -6x - 4\lambda = 0$$

$$F_y = 4y - 4\lambda = 0 \Rightarrow \lambda = y$$

$$F_\lambda = -(4x+4y-16) = 0$$
$$4x+4y-16=0$$

$$-6x-4(y)=0$$
$$4x+4y=16$$

$$-2x = 16$$
$$x = -8$$

$$4(-8)+4y=16$$
$$4y=48$$
$$y=12$$

$$f(-8,12) = 2(144) - 3(64)$$
$$= 96$$

$(-8,12,96)$