**MATH 109 – Final Exam** **SPRING 2026**

| Problem | Possible | Earned |
|---|---|---|
| 1 | 6 | |
| 2 | 6 | |
| 3 | 6 | |
| 4 | 6 | |
| 5 | 6 | |
| 6 | 6 | |
| 7 | 12 | |
| 8 | 12 | |
| 9 | 8 | |
| 10 | 16 | |
| 11 | 12 | |
| 12 | 16 | |
| 13 | 14 | |
| 14 | 6 | |
| 15 | 8 | |
| 16 | 6 | |
| 17 | 12 | |
| 18 | 6 | |
| 19 | 8 | |
| 20 | 6 | |
| 21 | 6 | |
| 22 | 8 | |
| 23 | 8 | |
| TOTAL POINTS | | 200 |

Name: _____

Instructor: _____

Section number: _____

Please check to make sure that your copy of the examination has all nine (8) pages including a cover sheet, 7 pages with Problems #1– #23.

Work in a neat and well-organized manner. Show your work on all problems. Indicate your answers clearly. Credit will not be given for an answer not supported by correct work.

The use of a scientific calculator TI-30XIIS will be permitted on this final examination. Any other calculator models are not allowed.

Cellphones, Headphones, Smartwatches and all electronic devices are PROHIBITED during the final exam.

**THIS EXAM IS SINGLE-SIDED.**




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[6] 1. a) Simplify: $3\sqrt{32}+7\sqrt{2}$

b) Rationalize the denominator: $\dfrac{3}{2\sqrt{6}}$

[6] 2. Subtract: $\dfrac{x}{x+3}-\dfrac{2}{x+4}$

[6] 3. Factor and simplify: $\dfrac{4x^2-25}{4x^2+8x-5}$

[6] 4. Find a **positive angle** less than $360^\circ$ or $2\pi$ radians that is **<u>coterminal</u>** with each of the given angle:

a) $1350^\circ$

b) $-\dfrac{13\pi}{5}$

[6] 5. Given $\vec{u}=5\vec{i}-2\vec{j}$ and $\vec{v}=-4\vec{i}+7\vec{j}$, find the following.

a) Find $\vec{u}-2\vec{v}$ .

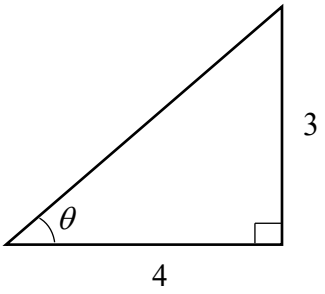
b) Find the dot product, $\vec{u}\cdot\vec{v}$ .

[6] 6. A central angle of $36^\circ$ in a circle of radius of 12 feet intercept an arc.

a) What is the **length** of the arc? Include units.

b) What is the **area of the sector** formed by the arc. Express the area in terms of $\pi$. Include units.

[12] 7. a) Find the **exact values** of **all** six trigonometric functions. The angle $\theta$ is an acute angle in the right triangle shown in the figure below.




(a) $\sin\theta =$ (b) $\csc\theta =$

(c) $\cos\theta =$ (d) $\sec\theta =$

(e) $\tan\theta =$ (f) $\cot\theta =$

b) Find the angle $\theta$ in degrees. Round to the nearest whole number.

[12] 8. Fill in each blank so that the resulting statement is true.

a) If $0^\circ < \theta < 90^\circ$, $\theta$ is a/an_____angle.

b) If $\theta = 90^\circ$, $\theta$ is a/an _____ angle.

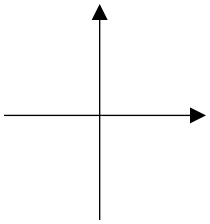
c) If $90^\circ < \theta < 180^\circ$, $\theta$ is a/an _____ angle.

d) If $\theta = 180^\circ$, $\theta$ is a/an _____ angle.

e) To convert degrees to radians, multiply degrees by _____.

f) To convert radians to degrees, multiply radians by _____.

[8] 9. Given $\tan\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find the **exact** values of the following:



a) $\sin\theta$

b) $1-2\cos^2\theta$

[16] 10. For the given angles below, find the quadrant in which the terminal side of the angle $\theta$ lies, its reference angle $\theta'$, and the **exact values** of each expression. No decimal answers.

a) $\theta = 330^\circ$

Quadrant:

$\theta' =$

$\cos(\theta') =$

$\cos(\theta) =$

b) $\theta = -135^\circ$

Quadrant:

$\theta' =$

$\sin(\theta') =$

$\sin(\theta) =$

c) $\theta = \frac{8\pi}{3}$

Quadrant:

$\theta' =$

$\tan(\theta') =$

$\tan(\theta) =$

d) $\theta = -\frac{13\pi}{6}$

Quadrant:

$\theta' =$

$\csc(\theta') =$

$\csc(\theta) =$

[12] 11. Determine the period, phase shift, and range of the following functions.

a) $y = -4\sin(3x-\pi)$

Amplitude:

Period:

Range:

b) $y = 2\sin\left(\frac{2}{3}x\right)+2$

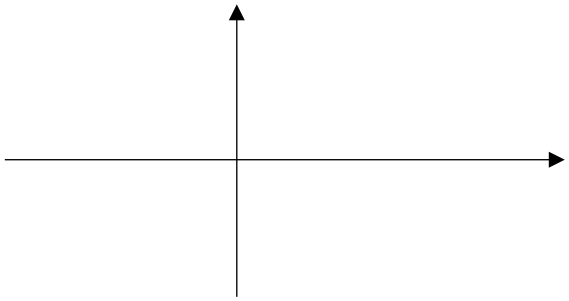
Amplitude:

Period:

Range:

[16] 12. Graph **<u>one period</u>** of each. The $x-$ axis must have **<u>at least 4 ticks labeled</u>** and the $y-$ axis **<u>at least 2</u>**.

a) $y=-2\cos(x)+1$

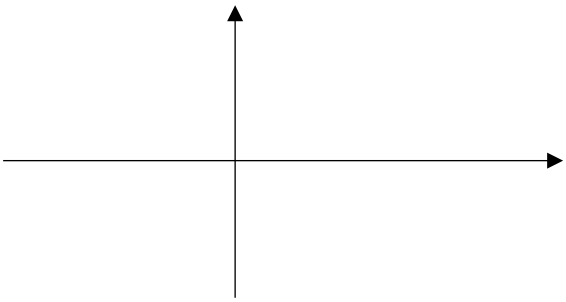


Amplitude:

Period:

Range:

b) $y=3\sin(2x)$



Amplitude:

Period:

Range:

[14] 13. True or False (Circle one)

a) Dot product of two vectors is a vector T or F

b) Period of the tangent function is $2\pi$ T or F

c) $\sec(-\theta)=\sec\theta$ T or F

d) Period of the cosine function is $2\pi$ T or F

e) $\sec\left(\frac{\pi}{5}\right)=\csc\left(\frac{3\pi}{10}\right)$ T or F

f) $\tan(-\theta)=-\tan(\theta)$ T or F

g) $\sin(\theta+\pi)=\sin\theta$ T or F

[6] 14. Find **<u>ALL</u>** solutions: $3\cos x-2=5\cos x-1$

[8] 15. Solve the equation **<u>in the interval</u>** $[0, 2\pi)$. $4\sin^2 x - 1 = 0$

[6] 16. Identity Completions (No partial credit):

a) $\tan^2 x + 1 =$ (Pythagorean identity)

b) $1 - \sin^2 x =$ (Pythagorean identity)

c) $\dfrac{\sin x}{\cos x} =$ (Quotient Identity)

[12] 17. Verify the identities:

a) $\cos x - \cos x \sin^2 x = \cos^3 x$

b) $\dfrac{\tan x + \sin x}{1 + \cos x} = \tan x$

[6] 18. State the **domain** and **range** of $y = \sin^{-1} x$, $y = \cos^{-1} x$, $y = \tan^{-1} x$ **in interval notation**.

| Function | Domain | Range |
|---|---|---|
| $y = \sin^{-1} x$ | | |
| $y = \cos^{-1} x$ | | |
| $y = \tan^{-1} x$ | | |

[8] 19. Find the following (**EXACT** values in radians.

a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

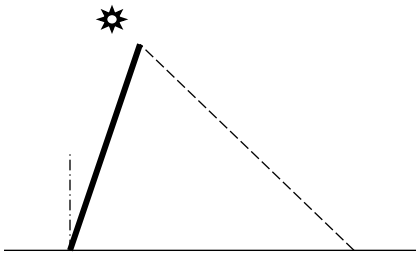
b) $\sin^{-1}\left(-\frac{1}{2}\right)$

c) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

d) $\tan^{-1}(1)$

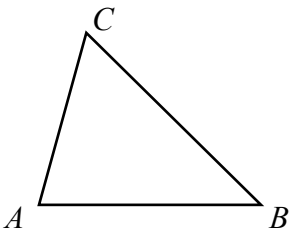
[6] 20. Solve $\tan\theta = -0.7$ in $[0, 2\pi)$. Round to **4 decimal places**.

[6] 21. A pole leans away from the sun at an angle of $9^\circ$ to the vertical. When the **angle of elevation** of the sun is $53^\circ$, the pole casts a shadow $47\ ft$ long on level ground. How long is the pole?
Round to the **nearest foot**.



[8] 22. Solve the triangle. Round **lengths** to the **nearest tenth** and **angle** measures to the **nearest degree.**

$a = 10.7\,ft$ , $c = 13.4\,ft$ , $C = 55^\circ$




[8] 23. a) Write the vector $\vec{u}$ in terms of $\vec{i}$ and $\vec{j}$ whose magnitude $\|\vec{u}\| = 15$ and the direction angle $\theta = 63^\circ$.

b) Write the vector $\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$ whose magnitude $\|\vec{v}\| = 20$ and the direction angle $\theta = 35^\circ$.

c) Find the **magnitude** of the **resultant vector** $\vec{u} + \vec{v}$. Round to the **nearest tenth**.

d) Find the **direction angle** of the **resultant vector** $\vec{u} + \vec{v}$ .Round to the **nearest degree**.