MATH 109 – Final Exam **SPRING 2026**

Problem	Possible	Earned
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	12	
8	12	
9	8	
10	16	
11	12	
12	16	
13	14	
14	6	
15	8	
16	6	
17	12	
18	6	
19	8	
20	6	
21	6	
22	8	
23	8	
TOTAL POINTS		200

Name: _____

Instructor: _____

Section number: Key

Please check to make sure that your copy of the examination has all nine (8) pages including a cover sheet, 7 pages with Problems #1– #23.

Work in a neat and well-organized manner. Show your work on all problems. Indicate your answers clearly. Credit will not be given for an answer not supported by correct work.

The use of a scientific calculator TI-30XIIS will be permitted on this final examination. Any other calculator models are not allowed.

Cellphones, Headphones, Smartwatches and all electronic devices are PROHIBITED during the final exam.

THIS EXAM IS SINGLE-SIDED.



109

[6] 1. a) Simplify: $3\sqrt{32}+7\sqrt{2}$

$= 3(4\sqrt{2})+7\sqrt{2} = 12\sqrt{2}+7\sqrt{2} = \boxed{19\sqrt{2}}$

b) Rationalize the denominator: $\frac{3}{2\sqrt{6}} = \frac{3}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{12}$

$= \boxed{\frac{\sqrt{6}}{4}}$

[6] 2. Subtract: $\frac{x}{x+3}-\frac{2}{x+4}$

$= \frac{x(x+4)-2(x+3)}{(x+3)(x+4)}$

$= \frac{x^2+4x-2x-6}{(x+3)(x+4)}$

$= \frac{x^2+2x-6}{(x+3)(x+4)}$

[6] 3. Factor and simplify: $\frac{4x^2-25}{4x^2+8x-5}$

$= \frac{(2x+5)(2x-5)}{(2x-1)(2x+5)}$

$= \boxed{\frac{2x-5}{2x-1}}$

[6] 4. Find a **positive angle** less than 360° or 2π radians that is **coterminal** with each of the given angle:

a) 1350°

$1350^\circ - 1080^\circ$

$= \boxed{270^\circ}$

b) $-\frac{13\pi}{5}$ $-\frac{13\pi}{5}+\frac{10\pi}{5}$

$= -\frac{3\pi}{5}+\frac{10\pi}{5}$

$\boxed{\frac{7\pi}{5}}$

[6] 5. Given $\vec{u}=5\vec{i}-2\vec{j}$ and $\vec{v}=-4\vec{i}+7\vec{j}$, find the following.

a) Find $\vec{u}-2\vec{v}$.

$5\vec{i}-2\vec{j}+8\vec{i}-14\vec{j}$

$= \boxed{13\vec{i}-16\vec{j}}$

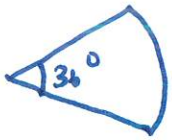
b) Find the dot product, $\vec{u}\cdot\vec{v}$.

$\vec{u}\cdot\vec{v} = 5(-4)+(-2)(7)$

$= -20-14$

$= \boxed{-34}$

[6] 6. A central angle of 36° in a circle of radius of 12 feet intercept an arc.



a) What is the **length** of the arc? Include units.

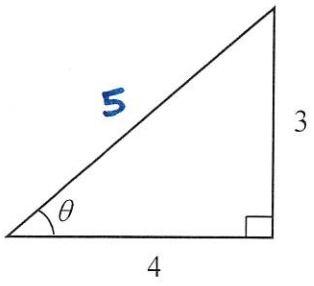
$36^\circ = 36^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{5}$ $\quad s = 12\left(\frac{\pi}{5}\right)$

$= \boxed{\frac{12\pi}{5}\text{ ft}}$

b) What is the **area of the sector** formed by the arc. Express the area in terms of π. Include units.

$A = \frac{1}{2}(12)^2 \cdot \frac{\pi}{5} = \frac{1}{2} \cdot 144 \cdot \frac{\pi}{5} = \boxed{\frac{72\pi}{5}\text{ ft}^2}$

[12] 7. a) Find the **exact values** of **all** six trigonometric functions. The angle θ is an acute angle in the right triangle shown in the figure below.



(a) $\sin\theta = \boxed{\frac{3}{5}}$ (b) $\csc\theta = \boxed{\frac{5}{3}}$

(c) $\cos\theta = \boxed{\frac{4}{5}}$ (d) $\sec\theta = \boxed{\frac{5}{4}}$

(e) $\tan\theta = \boxed{\frac{3}{4}}$ (f) $\cot\theta = \boxed{\frac{4}{3}}$

b) Find the angle θ in degrees. Round to the nearest whole number.

$\tan\theta = \frac{3}{4}$ $\quad \theta = \tan^{-1}(3/4)$

$\approx \boxed{37^\circ}$

[12] 8. Fill in each blank so that the resulting statement is true.

a) If $0^\circ < \theta < 90^\circ$, θ is a/an acute angle.

b) If $\theta = 90^\circ$, θ is a/an right angle.

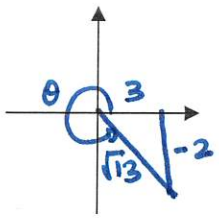
c) If $90^\circ < \theta < 180^\circ$, θ is a/an obtuse angle.

d) If $\theta = 180^\circ$, θ is a/an straight angle.

e) To convert degrees to radians, multiply degrees by $\frac{\pi}{180^\circ}$.

f) To convert radians to degrees, multiply radians by $\frac{180^\circ}{\pi}$.

[8] 9. Given $\tan\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find the **exact** values of the following:



a) $\sin\theta = \frac{-2}{\sqrt{13}} = \boxed{-\frac{2\sqrt{13}}{13}}$

b) $1-2\cos^2\theta$

$= 1-2\left(\frac{3}{\sqrt{13}}\right)^2$

$= 1-2\cdot\frac{9}{13} = 1-\frac{18}{13} = \boxed{-\frac{5}{13}}$

[16] 10. For the given angles below, find the quadrant in which the terminal side of the angle θ lies, its reference angle θ', and the **exact values** of each expression. No decimal answers.

a) $\theta = 330^\circ$



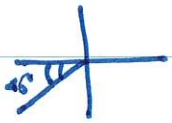
Quadrant: IV

$\theta' =$ 30°

$\cos(\theta') =$ $\frac{\sqrt{3}}{2}$

$\cos(\theta) =$ $\frac{\sqrt{3}}{2}$

b) $\theta = -135^\circ$



Quadrant: III

$\theta' =$ 45°

$\sin(\theta') =$ $\frac{\sqrt{2}}{2}$

$\sin(\theta) =$ $-\frac{\sqrt{2}}{2}$

$\frac{8\pi}{3} - \frac{6\pi}{3} = 2\pi/3$

c) $\theta = \frac{8\pi}{3}$



Quadrant: II

$\theta' =$ $\pi/3$

$\tan(\theta') =$ $\sqrt{3}$

$\tan(\theta) =$ $-\sqrt{3}$

d) $\theta = -\frac{13\pi}{6}$

$-\frac{13\pi}{6} + \frac{12\pi}{6} = -\frac{\pi}{6}$



Quadrant: IV

$\theta' =$ $\pi/6$

$\csc(\theta') =$ 2

$\csc(\theta) =$ -2

[12] 11. Determine the period, phase shift, and range of the following functions.

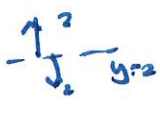
a) $y = -4\sin(3x-\pi)$

Amplitude: $|-4| = 4$

Period: $\frac{2\pi}{3}$

Range: $[-4, 4]$

b) $y = 2\sin\left(\frac{2}{3}x\right)+2$



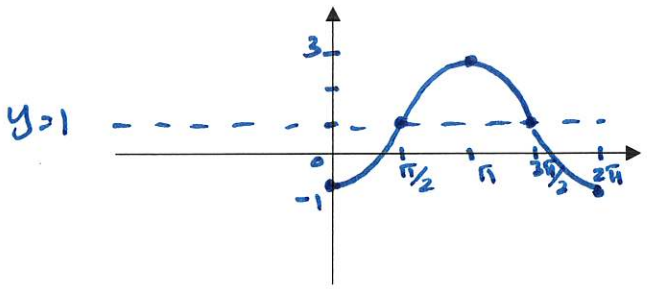
Amplitude: $|2| = 2$

Period: $\frac{2\pi}{2/3} = 3\pi$

Range: $[0, 4]$

[16] 12. Graph **one period** of each. The $x-$ axis must have **at least 4 ticks labeled** and the $y-$ axis **at least 2**.

a) $y=-2\cos(x)+1$

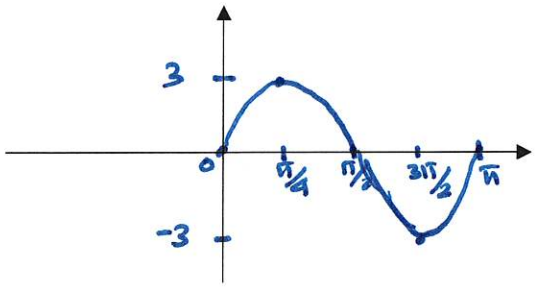


Amplitude: $|-2| = 2$

Period: 2π

Range: $[-1, 3]$

b) $y=3\sin(2x)$



Amplitude: $|3| = 3$

Period: $\frac{2\pi}{2} = \pi$

Range: $[-3, 3]$

[14] 13. True or False (Circle one)

a) Dot product of two vectors is a vector — T or (F)

b) Period of the tangent function is 2π — T or (F)

c) $\sec(-\theta)=\sec\theta$ — (T) or F

d) Period of the cosine function is 2π — (T) or F

e) $\sec\left(\frac{\pi}{5}\right)=\csc\left(\frac{3\pi}{10}\right)$ — (T) or F

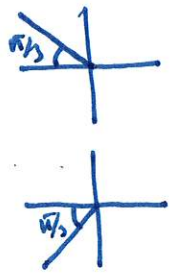
f) $\tan(-\theta)=-\tan(\theta)$ — (T) or F

g) $\sin(\theta+\pi)=\sin\theta$ — T or (F)

[6] 14. Find **ALL** solutions: $3\cos x-2=5\cos x-1$

$-2\cos x = 1$ ref∠ : $\pi/3$

$\cos x = -\frac{1}{2}$



$x = \frac{2\pi}{3} + 2n\pi$

$x = \frac{4\pi}{3} + 2n\pi$

n is an integer

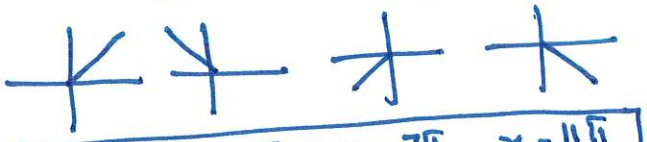
[8] 15. Solve the equation **in the interval** $[0, 2\pi)$. $4\sin^2 x - 1 = 0$

$4\sin^2 x = 1$

$\sin^2 x = 1/4$

$\sin x = \pm \frac{1}{2}$

$\sin x = \frac{1}{2}$ $\sin x = -\frac{1}{2}$ ref∠ : $\pi/6$



$x = \pi/6$ $x = \frac{5\pi}{6}$ $x = \frac{7\pi}{6}$ $x = \frac{11\pi}{6}$

[6] 16. Identity Completions (No partial credit):

a) $\tan^2 x + 1 =$ $\sec^2 x$ (Pythagorean identity)

b) $1 - \sin^2 x =$ $\cos^2 x$ (Pythagorean identity)

c) $\frac{\sin x}{\cos x} =$ $\tan x$ (Quotient Identity)

[12] 17. Verify the identities:

a) $\cos x - \cos x \sin^2 x = \cos^3 x$

LHS = $\cos x - \cos x \sin^2 x$

$= \cos x (1 - \sin^2 x)$

$= \cos x \cdot \cos^2 x$

$= \cos^3 x$ = RHS

b) $\frac{\tan x + \sin x}{1 + \cos x} = -\tan x$

LHS = $\frac{\tan x + \sin x}{1 + \cos x}$

$= \frac{\frac{\sin x}{\cos x} + \sin x}{1 + \cos x}$

$= \frac{\sin x + \sin x \cos x}{\cos x (1 + \cos x)}$

$= \frac{\sin x (1 + \cos x)}{\cos x (1 + \cos x)}$

$= \frac{\sin x}{\cos x}$

$= \tan x$ = RHS

[6] 18. State the **domain** and **range** of $y = \sin^{-1} x$, $y = \cos^{-1} x$, $y = \tan^{-1} x$ **in interval notation**.

Function	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

[8] 19. Find the following (**EXACT** values in radians.

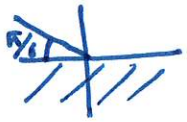
a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$



b) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$



c) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$



d) $\tan^{-1}(1) = \frac{\pi}{4}$



[6] 20. Solve $\tan\theta = -0.7$ in $[0, 2\pi)$. Round to **4 decimal places**.

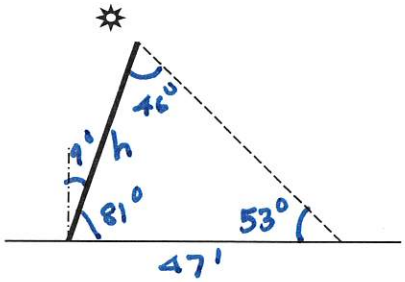
ref∠ : $\tan^{-1}(0.7) \approx 0.6107$



$x \approx 2.5309$

$x \approx 5.6725$

[6] 21. A pole leans away from the sun at an angle of 9° to the vertical. When the **angle of elevation** of the sun is 53°, the pole casts a shadow $47\ ft$ long on level ground. How long is the pole?
Round to the **nearest foot**.

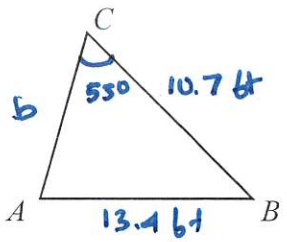


$$\frac{h}{\sin 53^\circ} = \frac{47}{\sin 46^\circ}$$

$h \approx 52$ ft

[8] 22. Solve the triangle. Round **lengths** to the **nearest tenth** and **angle** measures to the **nearest degree.**

$a = 10.7\,ft$, $c = 13.4\,ft$, $C = 55^\circ$ SSA



$\frac{\sin A}{10.7} = \frac{\sin 53^\circ}{13.4}$

$A \approx 40^\circ$ or ~~140°~~

only 1 Δ possible

$B \approx 87^\circ$

$\frac{b}{\sin 87^\circ} = \frac{13.4}{\sin 53^\circ}$

$b \approx 16.8$ ft

$A \approx 40^\circ$
$B \approx 87^\circ$
$b \approx 16.8$ ft

[8] 23. a) Write the vector $\vec{u}$ in terms of $\vec{i}$ and $\vec{j}$ whose magnitude $\|\vec{u}\| = 15$ and the direction angle $\theta = 63^\circ$.

$\vec{u} = (15\cos 63^\circ)\vec{i} + (15\sin 63^\circ)\vec{j}$

b) Write the vector $\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$ whose magnitude $\|\vec{v}\| = 20$ and the direction angle $\theta = 35^\circ$.

$\vec{v} = (20\cos 35^\circ)\vec{i} + (20\sin 35^\circ)\vec{j}$

c) Find the **magnitude** of the **resultant vector** $\vec{u} + \vec{v}$. Round to the **nearest tenth**.

$\vec{u} + \vec{v} = (15\cos 63^\circ + 20\cos 35^\circ)\vec{i} + (15\sin 63^\circ + 20\sin 35^\circ)\vec{j}$

$= 23.2\vec{i} + 24.8\vec{j}$ $\|\vec{u} + \vec{v}\| = \boxed{34.0}$

d) Find the **direction angle** of the **resultant vector** $\vec{u} + \vec{v}$. Round to the **nearest degree**.

$\tan\theta = \frac{24.8}{23.2}$ $\theta \approx \boxed{47^\circ}$