MATH 140 SAMPLE FINAL EXAM 2

Name_____ **Section**_____

Work in a neat and organized manner. All work must be shown on all problems. Full credit will not be given unless you have work to back up your answers, and your work is clearly shown. No credit will be given for numerical approximations to your answer without all the supporting work.

All work must be on these pages to be graded. If you use scratch paper, you MUST transfer your work to this test. A TI-30XIIS scientific calculator will be permitted on the final exam. **NO PHONES ALLOWED. *Anyone caught with any type of smart device will earn a grade of 0%.**

Consumer Surplus: $CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$ **Elasticity:** $E = \frac{-p * f'(p)}{f(p)}$

Producers Surplus: $PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$

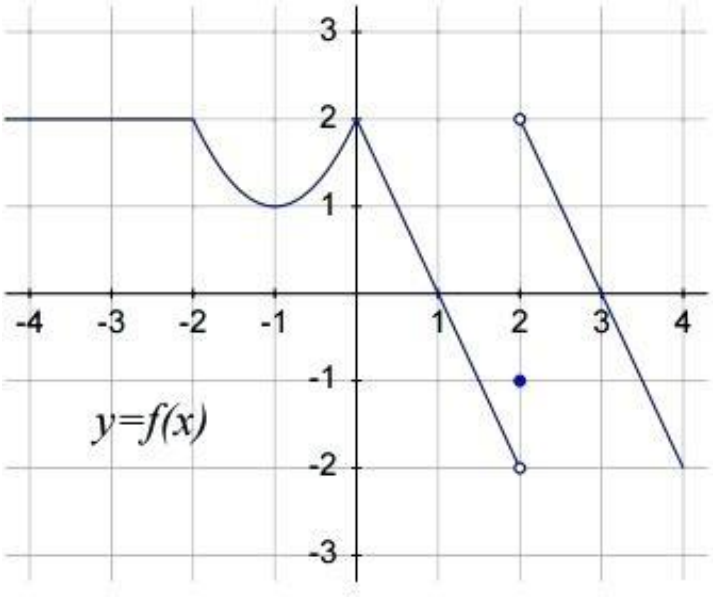
(15) 1. Calculate the following limits.

(a) $\lim_{x\to\infty} \frac{2x}{(x+3)^2-1}$

(b) $\lim_{x\to -1} (x^3+1)^2(x^4+5)$

(c) $\lim_{x\to 3} \frac{x^2-8x+15}{x^2+x-12}$

(8) 2. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a) $\lim_{x\to 2^-} f(x)$

(b) $\lim_{x\to 2^+} f(x)$

(c) $\lim_{x\to 2} f(x)$

(d) $\lim_{x\to -1} f(x)$

(28) 3. Compute the derivative of the following functions. **Do NOT** simplify.

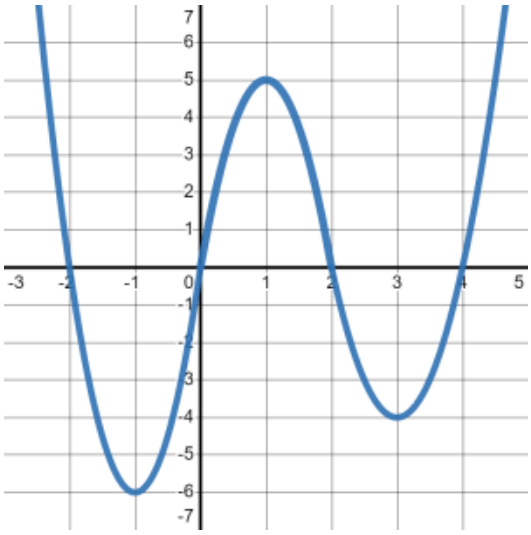
(a) $f(x) = \frac{8\sqrt{x}}{x+7}$

(b) $f(x) = \ln(x^4 + x - 6) + e^{-x}$

(c) $g(x) = \left(2e^{x^2} - x\right)\left(\frac{4}{3}x^2 + 6x - 10\right)$

(d) $h(x) = (x+2)^{1/3} + 2\ln x$

(8) 4. Referring to the given graph, list the intervals, if applicable, at which the derivative has the stated property. The points $(0, 0)$ and $(2, 0)$ are inflection points.



a) $f'(x)$ is positive

b) $f''(x)$ is positive

c) $f'(x)$ is negative

d) $f''(x)$ is negative

(8) 5.

A digital streaming service currently has **2,400 subscribers** paying **$30 per month**. Market research indicates that for every **$2 increase** in the monthly subscription price, the company will lose **100 subscribers**.

1. What monthly price should the company charge to maximize its monthly revenue?
2. What is the maximum monthly revenue?

(7) 6. Find the marginal cost at a production level of 1000 units if the cost function is given by

$$C(x) = 2\sqrt{x} + \frac{x^2}{8000} \ .$$

(9) 7. Find the producers' surplus for wedding cakes at a bakery at the equilibrium price for the given demand and supply functions:

$$D(x) = -\frac{4}{15}x + 15 \ , \ S(x) = \frac{1}{5}x + 1$$

(7) 8. Find dy/dx if $y^4 - 2x + 6x^2 = 5y + 6$

(7 pts) 9. Find dy/dx at (0,-1) if $y^2 = 5x^2 + e^x - 4x + 3$

(7 pts) 10.

A company's daily revenue R(t) (in thousands of dollars) from a new product launch is modeled by the function:

$$R(t) = 5 + 2t^2$$

where t is measured in days, with t = 0 representing the day of the launch. Find the **average daily revenue** of the company over the first 4 days.

(10)11. Let $f(x) = x^3 - 3x^2 - 9x + 5$. Use calculus to answer the following.

(a) Give the interval(s) in interval notation over which *f* is increasing. State relative extrema in (x,y) form.

(c) Give the interval(s) in interval notation over which *f* is concave down. State any inflection points in (x,y) form.

(21) 12. Compute the following integrals.

(a) $\int \left(12\sqrt[4]{x} - \frac{3}{x} + 50\right)\, dx$

(d) $\int_1^2 \frac{3x+2x^2}{x}\, dx$

(c) $\int (x^3 + 7)^4\, 3x^2\, dx$

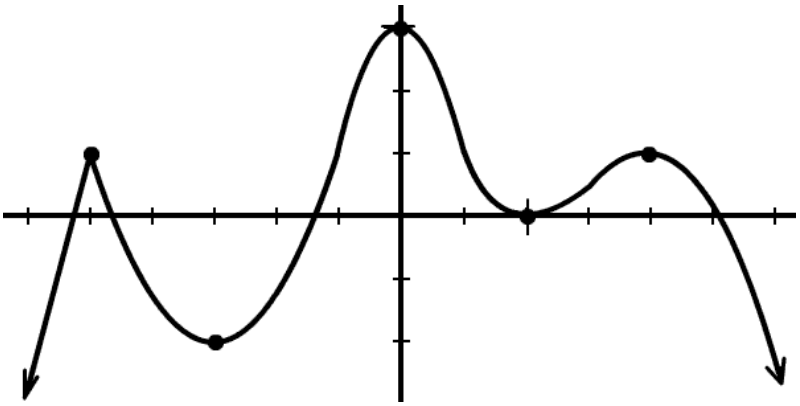
(7) 13.

A tech manufacturing company produces wireless earbuds. The total weekly cost C (in dollars) of producing x units is given by the cost function:

$$C(x) = 4,500 + 35x + 0.04x^2$$

Currently, the company produces $x = 500$ units per week, and management is increasing production at a rate of 25 units per week. How fast is the total weekly cost changing at this production level?

(9) 14. Use the graph of y=f(t) to answer the questions



a) Give interval(s) where $f'(t) < 0$

b) Give the t values for which $f'(t) = 0$

c) List the relative mimima

d) List the relative maxima

e) List the absolute maximum

f) List the absolute minimum

(7) 15.

The weekly demand function for a popular brand of wireless earbuds is given by:

$$q = f(p) = 500 - 10p$$

where q is the quantity demanded and p is the unit price in dollars ($0 < p < 50$).

a) Find the elasticity of demand function, $E(p)$.

b) Calculate the elasticity of demand when the price is set at $p = \$20$. Round your answer to two decimal places if necessary.

c) At $p = \$20$, is the demand **elastic**, **inelastic**, or **unitary**? Explain what this means for total revenue if the company decides to slightly raise the price.

16. (7) Find the equation of the tangent line to the graph of the function $f(x) = \frac{3}{x-1}$ at the point (2,3). State your answer in slope-intercept form.

17.

(7) 5. Ultra Mobile has costs that are given by

$$C(x) = 2000 + 90x + 0.2x^2$$

and price given by

$$p(x) = 210 - 0.3x$$

where x is the number of SIM cards produced. What is the number of SIM cards that Ultra Mobile must produce and sell in order to maximize profit? (Recall that the revenue is $R(x)=px$.) You must prove this using a derivative test.

18. (7) A research study finds that the concentration of a certain medication in a patient's bloodstream, C (in mg/L), t hours after it is administered, can be modeled by the function: $C(t) = 15\ln(t+1) - t$.

a) How many mg/L of medication is in the bloodstream after 3 hours? Round to 2 decimal places.

b) How fast is the amount of medication **changing** at 3 hours? Include units!

19. (7) According to the U.S. Census Bureau, the population of the United States can be approximated by

$$P(t) = 282.3e^{0.01t}$$

where P is in millions and t is the number of years since 2000.

Find the average value of the population from 2002 to 2006 (i.e. from $t = 2$ to $t = 6$).

20. (7) Graph a function which satisfies the following conditions:

$$\lim_{x\to-\infty} f(x) = 2, \quad \lim_{x\to 0} f(x) = \infty, \quad f' > 0 \ on\ (-\infty, 0) \cup (3, \infty); \quad f' < 0\ on\ (0,3)$$

$$f'' > 0\ on\ (-\infty, 0) \cup (0,5); \quad f'' < 0\ on\ (5, \infty)$$

21. (7) If $1000 is invested into a savings account with 6% compounded continuously ($A=Pe^{rt}$)

a) how long does it take for the investment to double? Show work algebraically.

b) how long does it take to reach $2600 (show all work <u>algebraically</u>)

c) how fast is the amount in the account increasing at 6 years? Include units.