MATH 140 **SAMPLE FINAL EXAM 1**

Name_____ Section_____

Work in a neat and organized manner. All work must be shown on all problems. Full credit will not be given unless you have work to back up your answers, and your work is clearly shown. No credit will be given for numerical approximations to your answer without all the supporting work.

All work must be on these pages to be graded. If you use scratch paper, you MUST transfer your work to this test. A TI-30XIIS scientific calculator will be permitted on the final exam. **NO PHONES ALLOWED.**
***Anyone caught with any type of smart device will earn a grade of 0%.**

Consumer Surplus: $CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$ **Elasticity:** $E = \frac{-p * f'(p)}{f(p)}$

Producers Surplus: $PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$

(24 pts) 1. Compute the derivative of the following functions. **Do NOT** simplify.

(a) $f(x) = \frac{e^{-x^3}}{x^4}$

(b) $f(x) = (x^2 + 1)\ln(3x^2 + x)$

(c) $h(x) = (2x^2 - 5x + 1)^{1/3}$

(d) $f(x) = \sqrt{2x+3} + \frac{5}{\sqrt[3]{x}} + 5$

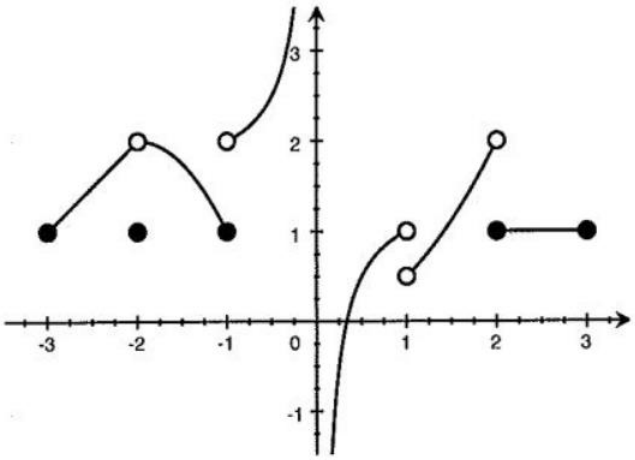
(8 pts) 2. Find the equation of the tangent line to $f(x) = \sqrt{4x+1}$ when $x = 0$. State your answer in slope-intercept form.

(10 pts) 3. Calculate the following limits. Show work algebraically.

(a) $\lim_{x \to 4} \frac{x^2+x-20}{x^2-10x+24}$

(b) $\lim_{x \to 5} \frac{\sqrt{x^2-16}}{x+1}$

(12 pts) 4. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a) $\lim_{x\to -1^-} f(x)$

(b) $\lim_{x\to 2^+} f(x)$

(c) $\lim_{x\to 2} f(x)$

(d) $\lim_{x\to -2} f(x)$

(e) $f(-1)$

(f) $f(1)$

(7 pts) 5. Suppose Arcteryx has costs that are given by $C(x) = x^2 + 2x$

and price given by $p(x) = 3x^3 - 4x^2 - 36x$

where x is the number of waterproof hiking bags produced.

a) Write the revenue function.

b) Write the profit function

c) Write the marginal profit function

(12 pts) 6. Given the function, $f(x) = x^3 + 6x^2$. Use calculus to answer the following.

a) Find the critical value(s) of $f(x)$.

b) Find the intervals where $f(x)$ is increasing and/or decreasing. State your answer in interval notation.

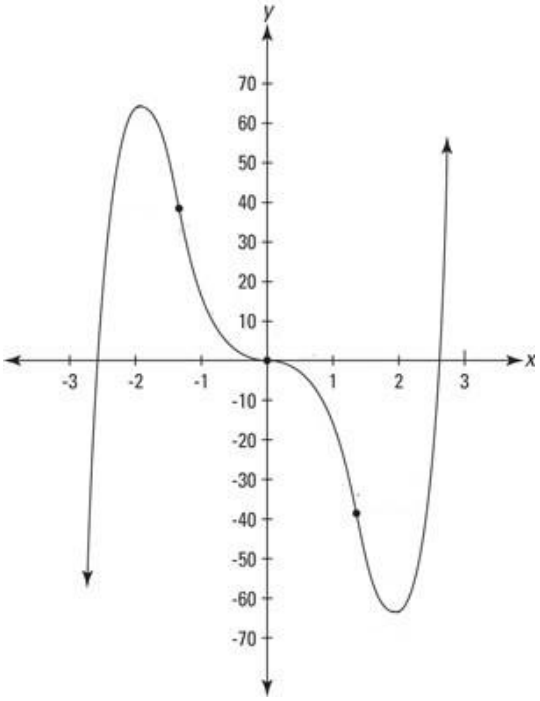
c) List the relative extrema in (x,y) form.

d) Find the intervals on which f(x) is concave up and concave down

e) Find all inflection points in (x,y) form.

(6 pts) 7. Blue Sky Vineyard determines the storage cost for storing *x* thousand cases of wine is determined by $C(x) = 12 + 9x - 3x^2 - x^3$ in thousands of dollars. How many cases of wine should optimally be stored in order to minimize inventory costs? What is the inventory cost at this minimum? Be sure to include correct units on your answers.

(6 pts) 8. Referring to the given graph of $f(x)$, answer the true/false questions below.



a) True or False: $f'(x)$ is positive at $x = -1$. _____

b) True or False: $f''(x)$ is positive on the interval $(2, \infty)$. _____

c) True or False: $f'(x)$ is negative on the interval $(-2, 0)$. _____

d) True or False: $f''(x)$ is negative at $x = 1$. _____

e) True or False: An inflection point exists at $x = 0$. _____

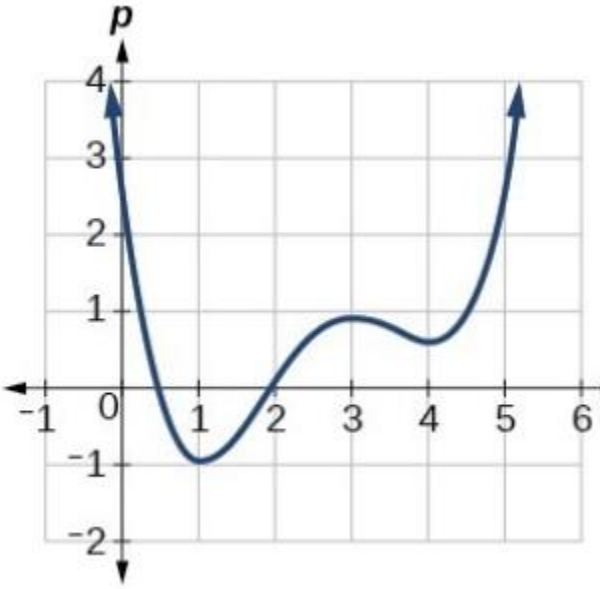
f) True or False: The point (1, -10) is a relative minimum. _____

(6 pts) 9. Communication and language are constantly changing. Glottochronology is a method used by linguists to investigate when related language became independent of each other. The percentage of two languages that agree (are identical) can be modeled by exponential decay as a function of time. The percentage of agreement between German and English can be model by $P(t) = 100e^{-0.00122t}$, where t is in years.

a) Find $P'(15)$. Round to 5 decimal places. Put units on your answer.

b) Explain what your answer means.

(12 pts) 10. Use the graph of y=f(t) to answer the questions



a) Give interval(s) where $f'(t) < 0$

b) Give the t values for which $f'(t) = 0$

c) List the relative mimima

d) List the relative maxima

e) List the absolute maximum

f) List the absolute minimum

(8 pts) 11. Sally's Sweets in NOLA finds that the marginal revenue, in dollars per pound, from the sale of *x* pounds of maple-coated pecans is given by $R'(x) = 0.1x + 0.0004x^3$.

a) If she knows she earned $180 for the sale of 10 pounds of pecans, find her total revenue function.

b) Using your answer from (a), determine how much revenue she earns from the sale of 100 pounds.

(7 pts) 12. Find dy/dx if $y^5 - 2y + 6x^2 = 5x + 6$

(6 pts) 13. Find dy/dx at (0,2) if $y^2 = 2x^3 + e^x - 3$

(10 pts) 14. The town council of a vacation town near Dollywood, Tennessee has determined that the demand and supply for rental cabins is

$$D(q) = 2000 - 2.5q \ , \ S(q) = 1520 + 5q$$

where q is the number of cabins and $D(q)$ and $S(q)$ are the rent in dollars per month, per cabin.

a) Find the equilibrium point.

b) Find the consumer surplus at the equilibrium point. Round your answer to the nearest dollar.

(6 pts) 15.

a) Write a piecewise function that gives the cost for x bags of mini candy bars. The cost is $5 a bag. However, if you buy more than 4 bags, they are $4.50 a bag.

b)Graph the function in part a). Mark endpoints as open or closed.

(28 pts) 16. Evaluate the following integrals. You can leave negative exponents, but don't leave fractions inside fractions. Leave exact answers (no rounded decimals).

(a) $\int \left(5\sqrt{x} + 10 - \frac{4}{x^3}\right) dx$

(b) $\int (2x - 5)(6x + 1) dx$

(c) $\int (t^3 - 1)^6 \, t^2 \, dt$

(d) $\int_0^1 x e^{x^2} dx$

(6 pts) 17. If $x^2 + y^2 = 10$, find dx/dt when x = 2, y= -3 and dy/dt = 5

(7 pts) 18. Suppose that for a company manufacturing flash drives, the cost and revenue equations are given by C=5000+ 2x and R = 10x-0.001x^2 where the production output in week 1 is x flash drives. If production is increasing at a rate of 500 flash drives per week when production is , find the rate of increase in profit in flash drives per week.

(6 pts) 19. Find the average value of $f(x) = \frac{2}{x} + 5$ on [1,5]

(6 pts) 20.The price-demand equation for hamburgers at a fast-food restaurant is x+400p=3000. Currently, the price of a hamburger is $4. If the price increased by 10%, will the revenue increase or decrease? By what percent?

(7pts) 21. An amusement park charges **$8** admission and averages **2,000** visitors per day. A survey shows that for each $1 increase in the admission price, 100 fewer people will visit the park. What price should the amusement park charge to maximize its daily revenue? Use calculus.