1. A dress sells regularly for \$138. It is on sale with the sale price being \$120. What is the percent of decrease of the sale price from the regular price rounded to the nearest tenth of a percent?

a) 87.0 % b) 66.7 % c) 13.0 % d) 15.0 % e) none of these

2. When the binary number 110110010 is converted to base 8, its base 8 representation is

a) 434 b) 626 c) 667 d) 662 e) none of these

3. If the complex number 1 + 2i is a zero of the polynomial P(x) = x^3 + a x + b with real values for a and b, than a equals

a) 1 b) 2 c) 3 d) 4 e) none of these

4. Evaluate $\log_1 3$

a) 3 b) 1/3 c) 1 d) 0 e) undefined

5. Determine the slope of the line passing through the points $(-5,6)$ and $(8,8)$.

a) $\frac{2}{13}$ b) $\frac{13}{2}$ c) $\frac{1}{4}$ d) 4 e) none of these

6. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos\theta = \frac{\sqrt{10}}{10}$ find the value of $\sin(2\theta)$.

a) $\frac{3}{10}$ b) $\frac{-3}{10}$ c) $\frac{3}{5}$ d) $\frac{-3}{5}$ e) none of these

7. If $x + \frac{1}{x} = -1$ then the value of $x^{2022} + \frac{1}{x^{2022}}$ equals

a) $(1 + \sqrt{5})/2$ b) $(-1 + i\sqrt{3})/2$ c) 4 d) 2 e) none of these

8. The local Starbucks has 15 employees and needs to select four of them to work opening shift on Christmas morning. Which of the following represents the number of ways this is possible?

a) $4 \cdot 3 \cdot 2 \cdot 1$ b) $15 \cdot 14 \cdot 13 \cdot 12$ c) $\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{4 \cdot 3 \cdot 2 \cdot 1}$

d) $\frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1}$ e) none of these

9. Consider all words (whether they actually occur in a language or not) that can be made from the letters a.b,c and e. A word can be rewritten by replacing any one instance of ab with e, by replacing any one instance of bc with e, or by deleting a single instance of e. In which of the following cases can you rewrite each word in the pair, in accordance with these rules, to get the same resulting word?

a) acac, aacc b) bc, cb c) cbcb,bcbc d) aeec, beec e) cbcb, cbab

10. The sum of two numbers a and b is three and their product is one.
Find $|a - b|$.

a) 2 b) $\sqrt{2}$ c) 0 d) $\sqrt{5}$ e) none of these

11. Which of the following numbers are divisible by 3 ?

a = $10^{33} + 1$; b = $10^{33} - 1$; c = $10^{33} + 2$; d = 10^{33}

a) a and b only b) b only c) b and c only d) c only e) all of them

12. Sixty-five percent of a barrel is filled with water. If twelve liters of water are added, water now occupies 80% of the barrel. How much water is in the barrel?

a) 52 liters b) 64 liters c) 77 liters d) 80 liters e) none of these

13. Compute: $\sin 25° (\tan 12.5° + \cot 12.5°)$

a) 2 b) 1 c) ½ d) $(\sqrt{3} + \sqrt{2})\ /4$ e) none of these

14. Find θ in the interval $[\ 0, \pi\ ]$ such that $\frac{3\ sec\theta+5}{2\ sec\theta+7} = 1$.

a) $\pi/6$ b) $3\pi/4$ c) $\pi/3$ d) $\pi/4$ e) none of these

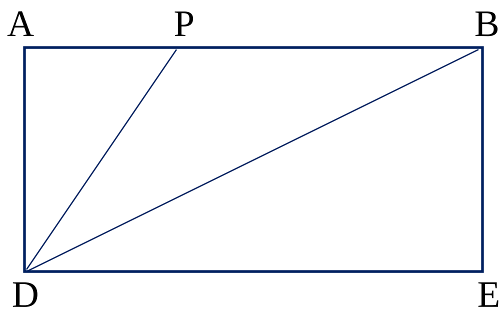
15, Let the sequence $\{\ a_n\ \}$ be defined by the relation $a_{n+2} + a_n = a_{n+1}$ for n $>$ 2 with $a_1 = 1\ \ and\ \ a_2 = 3$. Find the sum of $a_{2023} + a_{2024} + a_{2025}$

a) 2 b) 3 c) 4 d) 5 e) 6

16. A set of numbers is {4,9,11,15,21} is given. A sixth number is now added to this set and the mean increased by one. This new number is

a) 10 b) 12 c) 13 d) 18 e) none of these

17. Given the rectangle below the length of AD is one and segments DP and DB trisect angle ADE. What is the perimeter of triangle APD?



a) $3\sqrt{2}$ b) $2 + \frac{\sqrt{3}}{3}$ c) $2 + \sqrt{2}$ d) $\frac{3+\sqrt{3}}{2}$ e) $1 + \sqrt{3}$

18. F(x) is an even function if F(x) = F($-x$) and G(x) is an odd function if $G(-x) = -G(x)$. Let H(x) = $G\big(G(x)\big) \cdot F\big(F(x)\big)$. What can we conclude about H(x)?

a) No conclusion can be deduced b) H is neither even or odd

c) H is an even function d) H is an odd function

e) None of these are fully correct

19. When $x^{100} - 4x^{98} + 5x + 6$ is divided by $x^3 - 2x^2 - x + 2$ it has a remainder. Find the sum of the coefficients of this remainder.

a) 5 b) 6 c) 7 d) 8 e) none of these

20. Four integers a,b,c,d (not necessarily distinct) are chosen from the integers from one to 100 inclusive. All choices are independent. What is the probability that the value of ab minus cd is even?

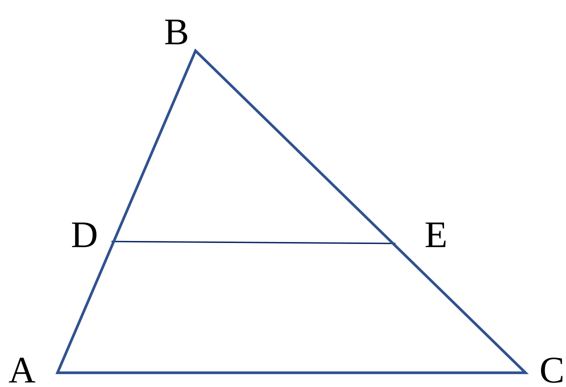
a) $\frac{5}{8}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{9}{16}$ e) none of these

21. What is the standard form of the equation of the circle which has the points $(-2,6)$ *and* $(2,-6)$ as the endpoints of its diameter?

a) $x^2 + y^2 = 160$ b) $x^2 + y^2 = \sqrt{40}$ c) $(x+2)^2 + (y-6)^2 = 40$

d) $x^2 + y^2 = 40$ e) none of these

22. Given triangle ABC as shown with segment DE parallel to side AC. If the ratio of the lengths of BE to EC is 4:3, and the area of trapezoid ADEC is 66, what is the area of triangle DBE?



a) 256/7 b) 32 c) 66 d) 48 e) none of these

23. If $1 + sin\theta = \frac{5}{9}, with\ \theta\ \epsilon \left[\frac{\pi}{2}, \pi\right]$ $then\ sin\theta - \cos\theta\ equals$

a) $\frac{1}{2}$ b) $\frac{-1}{3}$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$ e) none of these

24. A fair coin is tossed three times in succession. What is the probability of getting at least one show of heads?

a) 0.125 b) 0.25 c) 0.375 d) 0.5 e) none of these

25. What is the greatest number of points n in a plane, with the property that if S is a set of n points, then any subset of S is the intersection of S with a set defined by a linear inequality?

a) 2 b) 3 c) 4 d) 5 e) not enough information to determine

26. If $3^{a+2} = 9^b$ and $125^b = 5^{a-3}$, then what is the value of ab?

a) 60 b) -50 c) 66 d) -12 e) none of these

27. Let a and b denote the zeroes of F(x) = $\sqrt{2}\,(x+5)(x-8)$. Which of the following is true if b $<$ a ?

a) a + 13 = b b) a = b + $\sqrt{2}$ c) a + b = 13 d) a + b = 3 e) none of these

28. Simplify $\frac{1}{2+3i}$

a) $\frac{-2+3i}{13}$ b) $\frac{1}{2}+\frac{1}{3}i$ c) $\frac{1}{2}-\frac{1}{3}i$ d) $\frac{3+2i}{13}$ e) $\frac{2-3i}{13}$

29. Suppose three real numbers satisfy the equations x + y = 2 and xy $- z^2 = 1$
Find the value of x + y + z.

a) 2 b) 3 c) 4 d) 5 e) none of these

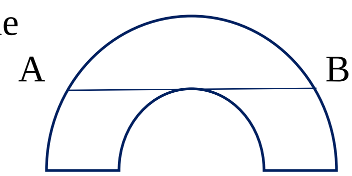
30. Evaluate $\sqrt{1.21} - \sqrt{0.01}$

a) $\sqrt{1.2}$ b) 1.09 c) 1 d) 0.9 e) none of these

31. Solve $\sqrt{3x+4} - \sqrt{x+5} = 1$

a) $\{-1,4\}$ b) $\{-1\}$ c) { 4 } d) { 2 } e) none of these

32. Given the two concentric semi-circles as shown. Chord AB of the larger circle is tangent to the smaller circle. Find the area between the two semicircles if the length of AB = $6\sqrt{5}$



a) 45π b) 22.5π c) 50π d) 48π e) none of these

33. Solve for x if $||x-1|-5| < 3$.

a) (5, ∞) b) $(-7,-1)$, (3,9) c) 5 d) (10,17) e) none of these

34. In the sequence: 16, 80, 48, 64, A, B, C, D each term beyond the second term is the arithmetic mean of the previous two terms. What is the value of D?

a) 59 b) 72 c) 64 d) 91 e) none of these

35. Which of the following is NOT equal to the value of the expression

$$\sqrt{\frac{4}{25}} + \sqrt{\frac{4}{25}} + \sqrt{\frac{4}{25}} + \sqrt{\frac{4}{25}}$$

a) $\sqrt{\frac{16}{100}}$ b) $\frac{8}{5}$ c) $4\sqrt{\frac{4}{25}}$ d) $\frac{56}{35}$ e) 160 %

36. If $4^{\frac{3x}{2}} + (2^x)^3 + 8^x + 2^{3x} = 2^{2023}$ then x equals

a) $\frac{2021}{4}$ b) $\frac{2021}{3}$ c) $\frac{2021}{2}$ d) 2023 e) none of these

37. If $F(x) = 3x^2 + 2x$ simplify the difference quotient for F(x) which is represented by

$$\frac{F(x+h)-F(x)}{h}$$

a) 6x+2 b) 6x + 3h + 2 c) 6x + 3h +2 + 4xh

d) 6xh + 6h + 2 e) none of these

38. Solve for the real values of x if $x^{\sqrt{x}} = x^{\frac{x}{2}}$

a) {1} b) {1,4} c) $\{-4\}$ d) { 3 } e) none of these

39. Suppose ABCD is a quadrilateral with perpendicular diagonals inscribed in a circle of radius 2. What is the value of $AB^2 + BC^2 + CD^2 + DA^2$?

a) 8 b) 16 c) 24 d) 32 e) none of these

40. Find the value of m such that $(x-1)(x+3)(x-4)(x-8) + \mathrm{m}$ is a perfect square.

a) 24 b) 32 c) 98 d) 196 e) none of these

41. What is the probability that a single card selected from a standard deck is a club?

a) 1/3 b) 2/3 c) 1/4 d) 1/13 e) none of these

42. When the expression $(x\sqrt{2} + y\sqrt{3}\,)^4$ is all multiplied out it becomes the expression: ax^4 + bx^3y + cx^2y^2 + dxy^3 + fy^4. The coefficient c is equal to

a) 6 b) 36 c) $8\sqrt{6}$ d) $12\sqrt{6}$ e) none of these

43. Calculate

$$\frac{1}{7\cdot 11} + \frac{1}{11\cdot 15} + \frac{1}{15\cdot 19} + \frac{1}{19\cdot 23} + \frac{1}{23\cdot 27}$$

a) $\frac{20}{189}$ b) $\frac{34}{189}$ c) $\frac{7}{621}$ d) $\frac{11}{621}$ e) $\frac{5}{189}$

44. Solve for x if $\log_{10} x = \ln(2x)$

a) $10^{\ln(2)/(1-\ln(10))}$ b) $\ln(\frac{e}{2})$ c) $\log(\frac{2}{e})$ d) $2^{\frac{e}{2}}$ e) none of these

45. The roots of $3x^4 + 2x + 3 = 0$ are $\{a, b, c, d\}$ Find the value of the expression $a^4 + b^4 + c^4 + d^4$.

a) 12 b) -4 c) -12 d) 4 e) none of these

46. Solve for the real value of x of $(\frac{1}{9})^x + (\frac{1}{6})^x = (\frac{1}{4})^x$

a) $log\,(\frac{1+\sqrt{5}}{2})$ b) 3.6 c) $\frac{log\sqrt{5}}{\log\ 1.5}$ d) $\frac{1+\sqrt{5}}{2}$ e) none of these

47. Suppose the function F satisfies the equation

$$F(xy) = F(x) + F(y) - 1$$

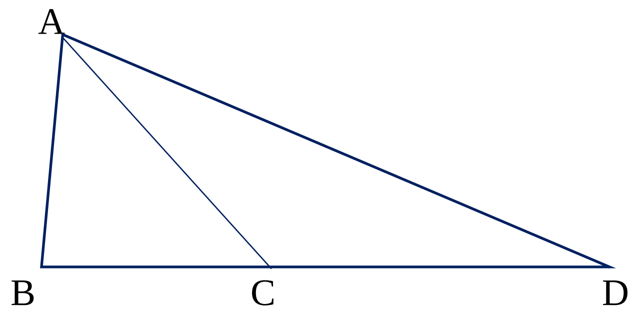
Suppose F(2) = 3. What is the value of F(8)?

a) 7 b) -3 c) 8 d) 2 e) none of these

48. Simplify: (10! −4) (10! + 4) − (10! −3) (10! + 3)

a) $(10!)^2 + 7$ b) $(10!)^2 - 7$ c) 7 d) -7 e) none of these

49. In the triangle below AC = CD and with angle ADC = 30° and angle BAC = 30°. If AB =12 what is the length of AD?



a) 18 b) $12\sqrt{3}$ c) $12\sqrt{2}$ d) $6\sqrt{3}$ e) none of these

50. For how many values of n does the series $\sum_{x=1}^{n} \frac{1}{x}$ have an integral sum?

a) none b) one c) two d) three e) infinitely many

51. How many real solutions exist to the equation $e^{3x} - 3e^{2x} - 4e^{x} + 12 = 0$?

a) 0 b) 1 c) 2 d) 3 e) none of these

52. If x = $\sqrt{2} + 1$ what is the value of $x^5 - 29x$?

a) 29 b) 73 c) 52 d) 12 e) none of these

53. From a cube of volume 27 cm^3 a cube of volume one is removed from the center of one of the edges. What is the surface area of the remaining solid?

a) 56 b) 58 c) 54 d) 48 e) none of these

54. Bubba's Landscaping has 60 yards of fencing with which to enclose a rectangular flower garden. If the garden is x yards long express the area A of the garden as a function of x

a) $x(30 - x)$ b) $x(60 - x)$ c) $x(20 - x)$ d) $x(10 - x)$ e) none of these

55. The numbers { a, b, c } form an arithmetic progression in the given order whose sum is 27. What is the value of b?

a) 6 b) 7 c) 9 d) 12 e) none of these

56. What is the value of $1 + \dfrac{1}{1 - \dfrac{1}{1 + \frac{1}{2}}}$?

a) 4.3 b) 4.5 c) 4 d) 6 e) none of these

57. For any two numbers a and b, define the operation * by

$$a * b = \frac{a}{b} + ab$$

What is the value of 20 * (4*2) ?

a) 202 b) 160 c) 42 d) 46 e) none of these

58. A bug is located in the xy plane at the point ($-10,4$). He then walks to the x axis and then to the point (5,4). What is the shortest possible distance he could have walked?

a) 7.5 $\sqrt{2}$ b) 15 $\sqrt{2}$ c) 17 d) 15 e) none of these

59. A red die and a green die are each numbered one through six and are rolled. Given that the sum was at least seven, what is the probability the green die showed a three?

a) $\frac{1}{7}$ b) $\frac{7}{12}$ c) $\frac{1}{6}$ d) $\frac{2}{9}$ e) none of these

60. Raquel has collected $3.80 in nickels and dimes. If she has a total of 62 coins how many dimes does she have?

a) 10 b) 12 c) 14 d) 48 e) none of these

61. Evaluate $\sqrt{9999^2 + 9999 + 10000}$

a) 10000 b) 9999 c) 7389 d) 20000 e) none of these

62. Let a and b be real numbers such that $a^2 + 4b^2 = 8ab$ with $0 < a < b$. Find the value of the expression $\frac{a+2b}{a-2b}$.

a) $-\sqrt{2}$ b) $-\sqrt{3}$ c) -2 d) $-\sqrt{5}$ e) none of these

63. If the vertices of a quadrilateral are given by the points A(0,0), B(5,0), C($2,2\sqrt{3}$), $D(1,3)$, and X is a point interior to the quadrilateral, what is the minimal value of the sum of the distances from X to all four vertices?

a) 9 b) 15 c) 18 d) 17 e) none of these

64. What is the shortest distance from the polar equation: r = $\frac{4}{sin\theta+cos\theta}$ and the polar point ($8\sqrt{2}$,$\frac{\pi}{4}$) ?

a) 0 b) $4\sqrt{2}$ c) $6\sqrt{2}$ d) 8 e) none of these

65. What is the length of the longest chord which can be drawn in the ellipse with equation $16y^2 + 36x^2 + 432\,x + 96y + 864 = 0$?

a) 12 b) 16 c) 36 d) $\sqrt{40}$ e) none of these

66. The roots of $x^2 + 2ax - 2b = 0$ are $\{a, b\}$. If a $\neq b$ $w$hat is the value of $a + b$?

a) 4 b) -4 c) 6 d) -2 e) none of these

67. If x is the solution to $3x + 5 = x + 13$, what is the value of $x^{\frac{-3}{2}}$?

a) 4 b) -4 c) -8 d) 0.125 e) none of these