

1. Suppose the graph of a line has a negative  $y$  intercept and a positive  $x$  intercept. If the slope is given by  $m$ , which of the following could be true?
  - i.  $m > 0$
  - ii.  $m < 0$
  - iii.  $m = 0$

a) i                      b) ii                      c) iii                      d) i and ii                      e) ii and iii
  
2. Let  $x$  and  $y$  be positive integers such that  $x^2 - y^2 = 2017$ . What is the value of  $x^2 + y^2$ ? (Hint: 2017 is a prime.)

a) 2025665              b) 71478863              c) 7160875              d) 2034145              e) 2045565
  
3. If the vertex of the parabola whose equation is  $y = x^2 + 8x + k$  is on the  $x$ -axis, what is  $k$ ?

a) 8                      b) 16                      c) 0                      d) 5                      e) 17
  
4. Suppose the sides of the two cubes differ by 2 and their volumes differ by 152. What is the length of the side of the large cube?

a) 4                      b) 6                      c) 9                      d) 7                      e) 5
  
5. Suppose  $x \leq y \leq z$  and  $x + y + z = 100$   
 $z - y = 10$   
 $y - x = 2$   
What is  $z$ ?

a)  $\frac{130}{3}$                       b) 50                      c) 17                      d)  $\frac{122}{3}$                       e) 48
  
6. A tetrahedron is a solid region enclosed by four equilateral triangles all with the same edge length. If the edge length is  $a$ , find a formula for the volume.

a)  $\frac{a^3}{24}\sqrt{11}$               b)  $\frac{a^3}{9}\sqrt{5}$               c)  $\frac{a^3}{12}\sqrt{2}$               d)  $\frac{a^2 + a}{7}$               e)  $\frac{5a^3\pi}{8}$

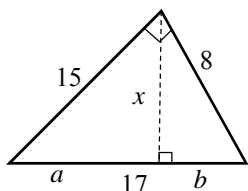
7. If  $x^{x^{x^{\dots}}} = 3$ . Then  $x =$

- a)  $\sqrt{3}$       b)  $\sqrt[3]{3}$       c) 3      d)  $\frac{1}{3}$       e) None listed.

8. If  $a = 1^2$ ,  $b = 2^2$ ,  $c = 3^2$ , ...,  $z = 26^2$ , find the product  $(a - x)(b - x)(c - x)\dots(z - x)$ .

- a)  $(26)!$       b)  $(26!)^2$       c)  $(26^2)!$       d) 0      e) None listed.

9. Find  $x$  if



- a)  $\frac{120}{17}$       b)  $\frac{23}{17}$       c)  $\frac{144}{17}$       d)  $\frac{144}{289}$       e) Can't be determined

10. Suppose  $x$  is a number so that  $x^2 + \frac{1}{x^2} = 62$ . What is the value  $x^4 + \frac{1}{x^4}$ ?

- a) 3844      b) 735      c) 3850      d) 3842      e) 756

11. For any positive integer  $n$ , suppose that  $\alpha$  is a solution to the equation

$$x^n + x^{n-1} + \dots + x + 1 = 0. \text{ Evaluate } \alpha^{n+1}$$

- a)  $\alpha$       b)  $\alpha^2$       c) 1      d) 2      e)  $-\alpha$

12. Suppose  $\frac{x+1}{y+1} = 2$ . Evaluate  $\frac{x+3}{y+2}$ .

- a) 1      b) 2      c)  $\frac{3}{2}$       d)  $\frac{2}{3}$       e) None listed.

13. Find the complex number  $z$  such that the reciprocal at  $z$  is the sum of the reciprocals of  $6 + 3i$  at  $6 - 3i$ .

- a)  $\frac{15}{4} + 0i$       b)  $3 - 6i$       c)  $3 + 6i$       d)  $9 + i$       e)  $18 + 2i$

14. If the ratio of  $a : b$  is 5:2 and the ratio of  $b : c$  is 3:5 what is the ratio of  $a : c$ ?

- a) 1:1      b) 3:2      c) 2:3      d) 5:6      e) None listed.

15. If  $\sqrt{4 + \sqrt{7}} - \sqrt{4 - \sqrt{7}}$  is simplified what is its value?

- a)  $\sqrt{3}$       b)  $\sqrt{2}$       c)  $2 + \sqrt{7}$       d)  $2 - \sqrt{7}$       e) None listed.

16. What is the value of  $\sin\left(\frac{2\pi}{5}\right) + \sin\left(\frac{4\pi}{5}\right) + \sin\left(\frac{6\pi}{5}\right) + \sin\left(\frac{8\pi}{5}\right)$ ?

- a)  $-1$       b)  $\frac{-1}{\sqrt{5}}$       c) 0      d)  $\frac{1}{\sqrt{5}}$       e) 1

17. Oil is pumped into a non-empty tank at a changing rate. The volume of oil in the tank doubles every minute and it is filled in 10 minutes. How many minutes did it take for the tank to be quarter full?

- a) 2      b) 5      c) 7      d) 8      e) 9

18. A carpenter saved \$2.50 in buying some lumber on sale. If he spent \$25 for the lumber, which of the following is closest to the percent he saved?

- a) 8%      b) 9%      c) 10%      d) 11%      e) 12%

19. Evaluate  $\sqrt{\sqrt{\dots\sqrt{3}}}$ .

- a)  $\sqrt{3}$       b)  $\sqrt[3]{3}$       c) 2      d) 1      e) 3.14159...

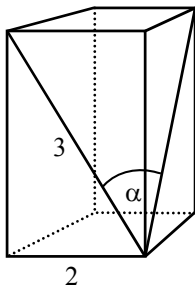
20. A function is defined by  $f(x) = |3 + 5^{3-x}| - 1$ . What is the range of  $f(x)$ ?

- a) All numbers    b)  $(2, +\infty)$ ,      c)  $[1, 3]$       d)  $[-1, +\infty]$       e)  $(0, +\infty)$

21. The value of expression  $(\tan 60^\circ + \tan 45^\circ)^2 - \sin 60^\circ$  is equal to

- a)  $(4 - 3\sqrt{3})/2$     b)  $(4 + 3\sqrt{3})/2$     c)  $(8 - 3\sqrt{3})/2$     d)  $(8 + 3\sqrt{3})/2$     e)  $\sqrt{3}/2$

22. A prism is based on a  $2 \times 2$  square. The diagonal of the side wall equals to 3. Find the tangent of the angle  $\alpha$  between two diagonals. (See the figure below.)



- a)  $\frac{\sqrt{5}}{2} \cdot 5/2$     b)  $\frac{\sqrt{14}}{2}$       c)  $\frac{2\sqrt{14}}{5}$       d)  $\frac{\sqrt{2}}{3}$       e)  $\sqrt{6}$

23. If 2,  $x$ ,  $y$ , 7 are four consecutive terms in an arithmetic progression, what is  $x$ ?

- a)  $\frac{17}{3}$       b)  $\frac{9}{2}$       c) 3      d)  $\frac{11}{3}$       e) None listed.

24. Solve for  $x$  if  $(\log x)(\log x) - \log x^7 + 12 = 0$ .

- a)  $\{10, 100\}$     b)  $\{10, 1000\}$     c)  $\{1000, 10000\}$     d)  $\{100, 10000\}$     e) None listed.

25. Which of the following does NOT represent a function?

- a)  $\{(x, y) \mid y = 5x - 2\}$   
b)  $\{(x, y) \mid x^2 + y^2 = 10, y \geq 0\}$   
c)  $\{(3,2), (4,2), (5,3), (6,4), (7,5)\}$   
d)  $\{(x, y) \mid y = e^x + 1\}$   
e)  $\{(x, y) \mid x^2 - y^2 = 10, x \geq 0\}$

26. Given  $f(x) = \frac{2x+3}{x-1}$  and  $g(x) = 2x-1$ , find  $f(g^{-1}(5))$ .

- a)  $\frac{9}{4}$     b)  $\frac{9}{2}$     c)  $\frac{13}{4}$     d) 9    e) 7

27. What is the remainder when  $6^{100}$  is divided by 7?

- a) 2    b) 3    c) 5    d) 1    e) 6

28. If  $\sin x + \cos x = 1.2$ , what is the value of  $\sin 2x$ ?

- a) .44    b) .5    c) .33    d) .34    e) Can't be determined

29. If  $\tan A + \tan B + \tan C = a$ ,  $0 < a < 10$ , and  $\angle A + \angle B + \angle C = 180^\circ$  then  $\tan A \tan B \tan C$ .

- a)  $\frac{1}{4}$     b)  $\frac{1}{2}a$     c)  $a$     d)  $a^2$     e) None listed.

30. Compute  $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+4+5+6}$ .

- a)  $\frac{17}{12}$       b)  $\frac{3}{4}$       c)  $\frac{5}{6}$       d)  $\frac{14}{15}$       e)  $\frac{12}{7}$

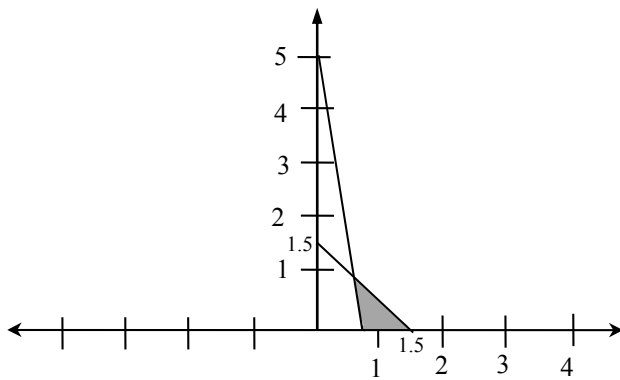
31. A right circular cone has diameter 16 cm and height 8 cm. It is sliced by a plane parallel to the base 2 cm from the apex. Find the volume of the resulting frustum.

- a)  $168\pi$       b)  $\frac{512\pi}{3}$       c)  $8\pi$       d)  $\frac{8\pi}{3}$       e) None listed.

32. The quadratic equation  $kx^2 + (k-1)x - 1 = 0$  has exactly one solution for

- a) any  $k$       b)  $k > 0$       c)  $k < 0$       d)  $k = -1$       e) no  $k$

33. The shaded region in the following diagram represents:



- |    |                  |                    |                    |                    |                 |
|----|------------------|--------------------|--------------------|--------------------|-----------------|
|    | $2y + 2x \leq 3$ | $2y + 2x \leq 3$   | $2y - 2x \leq 3$   | $2y - 2x \leq 3$   |                 |
| a) | $y + 7x \leq 5$  | b) $y + 7x \geq 5$ | c) $y - 7x \leq 5$ | d) $y - 7x \geq 5$ | e) None listed. |
|    | $x \geq 0$       | $x \geq 0$         | $x \geq 0$         | $x \geq 0$         |                 |
|    | $y \geq 0$       | $y \geq 0$         | $y \geq 0$         | $y \geq 0$         |                 |

34. The following system of equations

$$\begin{cases} 2x - 3y = 5 \\ -4x + 6y = -10 \end{cases}$$

- a) has no solutions
- b) has exactly one solution
- c) has exactly two solutions
- d) has exactly 4 solutions
- e) has infinitely many solutions

35. What is the value of  $\frac{x + x^2 + \dots + x^{10}}{x^{-3} + x^{-4} + \dots + x^{-12}}$ ?

- a)  $x^{22}$
- b)  $x^{13}$
- c)  $\frac{1+x}{1-x}$
- d)  $\frac{x^2}{1-x^2}$
- e)  $\frac{x^2}{1+x}$

36. If Sam gets a 97 on his next exam, his exam average will be 90. If instead of the 97, Sam gets a 73 on the next exam, then his exam average will be 87. How many exams has he taken?

- a) 8
- b) 5
- c) 10
- d) 6
- e) 7

37. An equilateral triangle with sides of 10 cm is inscribed in a square  $ABCD$  such that one vertex is at  $A$ , another vertex on  $\overline{BC}$  and one on  $\overline{CD}$ . Find the area of the square.

- a)  $25(2 + \sqrt{3})$
- b)  $25(2 - \sqrt{3})$
- c)  $\frac{100}{2 + \sqrt{3}}$
- d)  $\frac{100}{2 - \sqrt{3}}$
- e) None listed.

38. The matrix product  $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$  is equal to

- a)  $\begin{bmatrix} 5 & 4 \\ 9 & 0 \end{bmatrix}$
- b)  $\begin{bmatrix} 1 & 4 \\ 9 & 0 \end{bmatrix}$
- c)  $\begin{bmatrix} 7 & 2 \\ 6 & 0 \end{bmatrix}$
- d)  $\begin{bmatrix} 7 & 2 \\ 3 & 6 \end{bmatrix}$
- e) None listed.

39. Suppose  $\beta = \frac{1}{1 + \frac{1}{1 + \dots}}$ . Evaluate  $\beta^2 + \beta$ .

- a) 1                      b) 2                      c)  $\frac{1}{2}$                       d)  $\beta - 1$                       e) 0

40. A thin string has length 1 meter. Somebody cut off  $\frac{2}{5}$  of its length. Is the remaining piece of the string sufficiently long to wrap around a pole of diameter 20 cm?

- a) Yes      b) No      c) Not enough information.      d) Too much contradictory information.      e) None listed.

41. Let  $f(x)$  be a function satisfying  $f(x) + 2f\left(\frac{1}{x}\right) = 3 + x$  for all non-zero real number  $x$ .

What is  $f(2)$ ?

- a)  $\frac{1}{3}$                       b)  $\frac{2}{3}$                       c) 1                      d)  $\frac{4}{3}$                       e) None listed.

42. The roots of  $ax^2 + bx + c = 0$  are the squares of the roots of  $x^2 + x + 1$  then  $(a, b, c)$  equals

- a) (0,1,1)                      b) (1,1,0)                      c) (1,0,1)                      d) (1,1,1)                      e) None listed.

43. Consider the equation  $x^7 - 6x^6 - 12x^5 + 200x^4 - 720x^3 + 1248x^2 - 1088x + 384 = 0$ . It has 2 as a root of multiplicity 6. What is the other root?

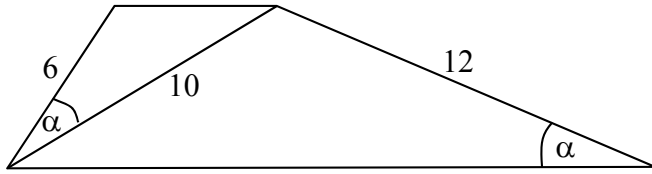
- a) 2                      b) 17                      c) -2                      d) -6                      e) Can't tell with information given

44. What is the value of  $\frac{\log_{10} 20 \cdot \log_{20} 144}{\log_{10} 11 \cdot \log_{11} 12}$ ?

- a) 4                      b) 2                      c)  $\frac{240}{11}$                       d) 5                      e) 6



45. The figure below represents a trapezoid. Notice that the two indicated angles  $\alpha$  are equal. Find the length of the longer base of the trapezoid.



The length of the base is:

- a) 18                      b) 20                      c) 22                      d) 24                      e) 28
46. Compute  $\frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \dots$
- a)  $\frac{9}{19}$                       b)  $\frac{5}{8}$                       c)  $\frac{7}{9}$                       d)  $\frac{3}{4}$                       e) 5
47. How many  $a$ 's exist such that  $a$  is between 1 and 20; and the remainders when  $a$  is divided by 2 and 3 are 1 and 2, respectively.
- a) 0                      b) 1                      c) 2                      d) 3                      e) 4
48. What is the base 6 representation of the sum  $0.1_2 + 0.1_3$  ?
- a)  $0.1_6$                       b)  $0.2_6$                       c)  $0.3_6$                       d)  $0.4_6$                       e)  $0.5_6$
49. A dodecahedron is a solid region enclosed by 12 pentagons all the small size. How many edges does a dodecahedron have?
- a) 30                      b) 25                      c) 40                      d) 32                      e) 15

50. How many vertices does a dodecahedron have?

- a) 25                      b) 18                      c) 20                      d) 23                      e) 12

51. Compute  $\sum_{n=1}^{24} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ .

- a) 4                      b)  $3 + 2\sqrt{2}$                       c)  $\sqrt{24}$                       d)  $\sqrt{24} - \sqrt{23}$                       e)  $\sqrt{24} + \sqrt{23}$

52. If the angles are in radians, let  $x = \sin 1$ ,  $y = \sin 2$ ,  $z = \sin 3$ . Which of the following is true?

- a)  $x < z < y$                       b)  $x < y < z$                       c)  $y < z < x$                       d)  $y < x < z$                       e)  $z < x < y$

53. In a right triangle, the ratio of the difference of the lengths of the legs to the length of the hypotenuse is  $1/2$ . Calculate the tangents of the acute angles. The greater value of these two is:

- a)  $\frac{1}{2}$                       b) 2.714                      c) 31                      d)  $\frac{\sqrt{2}}{2}$                       e)  $\frac{4 + \sqrt{7}}{3}$

54. The sum of the third and fourth terms of a sequence of consecutive integers is 47. What is the sum of the first five terms?

- a) 120                      b) 125                      c) 100                      d) 115                      e) None listed.

55. Let  $a$  and  $b$  be positive real numbers with  $a + b = 4$ . What is the minimum value of

$$\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right)?$$

- a) 2                      b)  $\frac{8}{3}$                       c)  $\frac{9}{4}$                       d) 3                      e) 4

56. If the set of four numbers  $\{4, 7, 8, x\}$  is supplemented by the number 2, the arithmetic average of the set will be increased by 2. Therefore

- a)  $x = -51$       b)  $x = -6$       c)  $x = 10$       d)  $x = 29$       e)  $x = 0$

57. If  $a \neq b$  then for how many distinct pairs of positive integers does  $a^b = b^a$ ?

- a) Infinitely many    b) 1      c) 4      d) 9      e) 2

58. Let the polynomial  $p(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$  satisfy  $p(0) = p(1) = p(2) = p(-1) = 0$ , and  $p(-2) = 12$ . Then  $p(3)$  equals

- a)  $\frac{1}{3}$       b)  $-\frac{1}{2}$       c) 6      d) 12      e) 24

59. The lower two vertices of a square lie on the  $x$ -axis and the upper two lie on the curve of  $y = 15 - x^2$ . What is the area of the square?

- a) 9      b)  $10\sqrt{2}$       c) 16      d) 25      e) 36

60. The last digit of  $777^{777}$  is

- a) 1      b) 3      c) 5      d) 7      e) 9

61. For how many integers  $m$ ,  $10 \leq m \leq 100$ , is  $m^2 + m - 90$  divisible by 17?

- a) 6      b) 8      c) 14      d) 22      e) 10

62. To write down all numbers 1 to  $n$ , inclusive, it is necessary to use 2883 digits. What is  $n$ ?

- a) 100                  b) 1000                  c) 897                  d) 997                  e) 1897

63. A square and an equilateral triangle have equal perimeters. The square has area 36 square units. What is the length of an altitude of the triangle?

- a)  $8\sqrt{3}$                   b)  $6\sqrt{3}$                   c)  $2\sqrt{3}$                   d)  $4\sqrt{3}$                   e) None listed.

64. What is the value of the sum  $\sum_{k=1}^{1000} (-1)^{k!}$  ?

- a) 0                  b) 15                  c) 82                  d) 1002                  e) 998

65. Find all values of  $a$  such that  $x^2 + (a - 3)x + a = 0$  has two distinct positive roots.

- a)  $(-\infty, 1)$                   b)  $(9, +\infty)$                   c) 7                  d)  $0 < a < 1$                   e) 16

66. If  $2^x + \frac{32}{2^x} = 12$ , what is the sum of the roots?

- a) 3                  b) 2                  c) 6                  d) 8                  e) 5

67. A quarter of a quarter is

- a)  $\frac{1}{2}$                   b)  $\frac{1}{4}$                   c)  $\frac{1}{8}$                   d)  $\frac{1}{16}$                   e)  $\frac{1}{44}$