1. The number $\left(\sqrt{2-\sqrt{3}}-\sqrt{2+\sqrt{3}}\right)^2$ is equal to

a) 2 b) 3 c) 4 d) $\sqrt{3}$ e) $2\sqrt{3}$

2. Consider the equation $3y + 4x = 10$. The slope of this line is

a) 4/3 b) $-4/3$ c) $-3/4$ d) 3/4 e) None listed.

3. Which of these numbers does **not** belong to the set of solutions of the inequality $(x^4+1)(2-x)>0$?

a) -3 b) -1 c) 0 d) 1 e) 3

4. Numbers {a, b, c} in this order form an arithmetic progression. Their sum is 27. What is the value of b?

a) 0 b) 1 c) 7 d) 2 e) 9

5. A triangle of sides 6 and 10 and the angle between them 120° is inscribed in a circle. What is the radius of the circle?

a) 7 b) 14 c) 28 d) $\frac{14\sqrt{3}}{3}$ e) $\frac{28\sqrt{3}}{3}$

6. The following system of equations $\begin{cases}3x-2y=6\\4y-6x=12\end{cases}$

a) has no solutions.
b) has exactly one solution.
c) has exactly two solutions.
d) has exactly four solutions.
e) has infinitely many solutions.

7. How many $a's$ exist such that a is between 1 and 50; and the remainders when a is divided by 2, 3, and 5 are 1, 2, 3, respectively.

a) 0 b) 1 c) 2 d) e) 4

8. Suppose $\beta = \dfrac{1}{2+\dfrac{1}{2+...}}$. Evaluate $\beta + 2$.

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) β d) $\frac{1}{\beta}$ e) None listed.

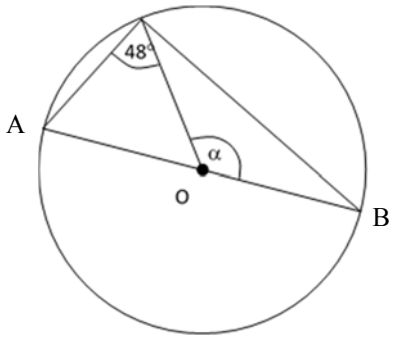
9. Compute $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}...$

a) 1 b) $\frac{3}{2}$ c) $\sqrt{2}$ d) 2 e) None listed.

10. Suppose $ij = k$, $jk = i$, $ki = j$, $i^2 = j^2 = k^2 = 1$. For any triple $a, b,$ and c, the rule $(ab)c = a(bc)$ is allowed, ji must equal

a) -1 b) 0 c) k d) $-ij$ e) None listed.

11. If *AB* is the diameter of the circle shown below, and O is its center, what is the angle α?



a) α= 138° b) α= 96° c) α = 112° d) α= 72° e) α= 74°

12. Let $n = 1 \times 3 \times 5 \times 7 \times \ldots \times 2017$ (so n is the product of the odd integers from 1 to 2017). What is the last digit of n?

a) 1 b) 3 c) 5 d) 7 e) 9

13. How many real numbers x with $0 < x \leq 10$ are solutions to $\log_{10}(x) = \sin(x)$, where x in $\sin(x)$ is in radians?

a) 0 b) 1 c) 2 d) 3 e) 4

14. Suppose $f(x)$ is a function that satisfies $f(x) + 5f\left(\frac{1}{x}\right) = 3 + x$ for all non-zero real numbers x. What is $f(4)$?

a) $\frac{5}{4}$ b) 1 c) 2 d) $\frac{27}{83}$ e) $\frac{37}{96}$

15. Suppose y is a real number such that $2 < y < 3$ and $y^3 - 4y - 5 = 0$. What is the closest integer to y^2?

a) 4 b) 5 c) 6 d) 7 e) 8

16. If a is a positive real number, what is the area of the region in the first quadrant that is bounded above by the graph of $y = x$ and below by the graph of $y = 2|x - a|$?

a) $\frac{2a^2}{3}$ b) a^2 c) $\frac{4a^2}{3}$ d) $2a^2$ e) None listed.

17. A fair coin is tossed repeatedly. What is the probability that we obtain a total of two tails before we obtain a total of three heads?

a) $\frac{3}{16}$ b) $\frac{9}{16}$ c) $\frac{5}{8}$ d) $\frac{11}{16}$ e) $\frac{3}{4}$

18. Find a positive value k such that $x^3 - 4x + k = 0$ has exactly two distinct real solutions.

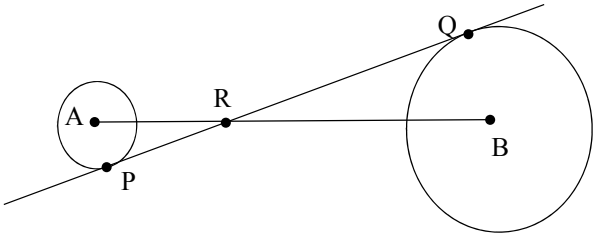
a) 3 b) $15\sqrt{5}/8$ c) 8 d) $16\sqrt{3}/9$ e) $2\sqrt{2}$

19. Which of the following are factors of $x^6 - 1$?

I. $x + 1$
II. $x^2 + 1$
III. $x^2 + x + 1$

a) I only
b) I and II only
c) II and III only
d) I and III only
e) I, II, and III

20. The line $\overleftrightarrow{PQ}$ is tangent to a circle of radius 1 centered at A and a circle of radius 3 centered at B with points of tangency P and Q, respectively. The segment $\overline{PQ}$ intersects $\overline{AB}$ at R. If $PQ = 6$, then AR equals



a) $\sqrt{13}/2$

b) $\sqrt{5}/2$

c) $\sqrt{52}/3$

d) $\sqrt{20}/3$

e) Cannot be determined from the given information.

21. If x and y are simultaneous solutions of the system of equations
$2x - y = 3$
$x + 2y = 7$ then $x + y$ equals

a) 3 b) 10/3 c) 11/5 d) 24/5 e) None listed.

22. Given positive real numbers x and y satisfying the equation $\frac{y^2}{x^2} + \frac{x^2}{y^2} = 2$, what is the value of $\frac{y}{x} + \frac{x}{y}$?

a) $\frac{1}{\sqrt{2}}$ b) 1 c) $\sqrt{2}$ d) 2 e) None listed.

23. Find x if $66_8 + 132_4 + 1011_2 = x_8$ where the subscript denotes the base the number is written in.

a) 216 b) 137 c) 75 d) 115 e) None listed.

24. If $\dfrac{3^{2018} - 3^{2017}}{54} = 3^n$, then what is the value of n?

a) 2015 b) 2014 c) 2012 d) 2013 e) 2000

25. Let a, b, c, d and e be numbers such that the average of a and b is 8 and the average of c, d and e is 44. What is the average of a, b, c, d and e?

a) 26 b) 148/6 c) 23 d) 148/5 e) 30

26. Find all value(s) of x, in $[0, 2\pi]$ such that $\sqrt{3}\sin x + \cos x = \sqrt{2}$?

a) $\pi/12$ and $7\pi/12$
b) $\pi/12$ and $13\pi/12$
c) $\pi/12$
d) Cannot be determined.
e) None listed.

27. If $a + b = 1$ and $a^2 + b^2 = 7$, what is the value of $a^3 + b^3$?

a) 2 b) 10 c) 7 d) 16 e) –21

28. If $n = 1111_2$. what is the value of n^2 in base 2?

a) 10001001_2 b) 10101010_2 c) 100001_2 d) 1111111_2 e) 11100001_2

29. Suppose that the sum of two numbers is 10 and that the sum of their quotient and its reciprocal is 6. What is the product of the two numbers?

a) 12 b) 13 c) 16 d) 12.5 e) None listed.

30. Solve for *x* if $(x+1)(x+2)(x+3) = (x-3)(x+4)(x+5)$.

a) 11/3 b) 4 c) –3 d) –11/4 e) –11/3

31. If the third term of an arithmetic progression is 7 and the seventh term is 3, what is the tenth term?

a) 10 b) 5 c) 0 d) 6 e) None listed.

32. Let *x* and *y* be two positive numbers such that $x^2 - y^2 = 2xy$. What is the value of *x*/*y*?

a) 3 b) $\sqrt{2}$ c) $\sqrt{3/5}$ d) $\sqrt{3}$ e) $1+\sqrt{2}$

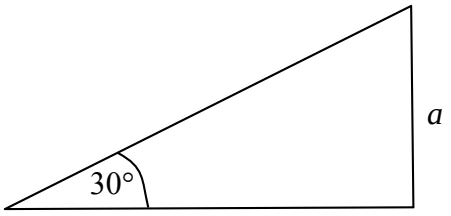
33. $\sin^2\theta + \sin^2(\theta - 90^\circ) =$

a) 0 b) 1 c) Depends upon θ d) Too little information. e) None listed.

34. Solve the value of *x* if $9^9 + 9^9 + 9^9 = 9^{2x+1}$.

a) 4 b) 4.25 c) 4.5 d) 4.75 e) None listed.

35. What is the perimeter of the orthogonal triangle shown below?



a) $\left(2+\sqrt{2}\right)a$ b) $\left(2+\frac{\sqrt{2}}{2}\right)a$ c) $\left(3+\sqrt{3}\right)a$ d) $\left(3+\frac{\sqrt{3}}{2}\right)a$ e) $\left(3+\frac{\sqrt{2}}{2}\right)a$

36. If the set of three numbers {3, 7, x} is supplemented by the number 1, the arithmetic average of the set will be decreased by 1. Find number x. In which of these groups does it appear?

a) $\{-2,-3,-7\}$ b) $\{-1,-5,18\}$ c) $\{1,4,5\}$ d) $\left\{\sqrt{2},\sqrt{3},\ln 2\right\}$ e) $\{0,-10,-100\}$

37. Sally wants to fence in a rectangular enclosure with the largest possible area. If one side is bounded by a river which does not need a fence and she has material for 22 more feet of fence for the other three sides, what is the length in feet of the diagonal of the rectangular enclosure that she can make with the largest area?

a) 13 b) $11\sqrt{5}/2$ c) $22\sqrt{2}/3$ d) $2\sqrt{34}$ e) None listed.

38. If $A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$ what is the product of AA^T where A^T is the transpose of A?

a) $\begin{bmatrix}13 & 11\\ 11 & 17\end{bmatrix}$ b) $\begin{bmatrix}20 & 10\\ 10 & 10\end{bmatrix}$ c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ d) $\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$ e) None listed.

39. On a table are two identical boxes. Box A has 3 red and 2 green marbles and box B has 1 red and 3 green marbles. A box was chosen at random and a green marble was drawn. What is the probability the marble came from box A?

a) $\frac{1}{5}$ b) $\frac{1}{2}$ c) $\frac{8}{23}$ d) $\frac{5}{9}$ e) None listed.

40. A rectangular box has dimensions 8 by 9 by 12 (all dimensions in inches). What is the length of the longest straight stick which will fit completely inside the box?

a) 17 inches b) 12 inches c) 18 inches d) 15 inches e) None listed.

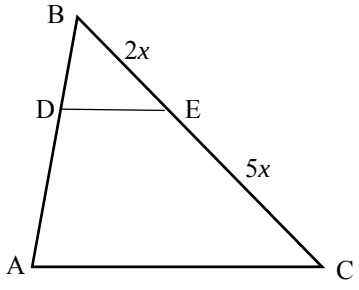
41. $F(x) = ((x+4)^5(x-9)^6(2x+5)^4(11x-3)^7)^2$ is graphed. How many times does the graph of $F(x)$ completely cross the x-axis?

a) 66 b) 2 c) 12 d) 36 e) 0

42. Suppose $\log_{10} x = 4$ and $\log_{10}(xy) = 6$. What is the value of y^3?

a) 10^7 b) 10^6 c) 10371 d) $10^{3/2}$ e) Can't be determined.

43. Consider triangle ABC with line DE parallel to AC. If the length of BE is $2x$, the length of EC is $5x$ and the area of the triangle BDE is 16, what is the area of the trapezoid ADEC?



a) 49/4 b) 49/360 c) 360/49 d) 90 e) None listed.

44. Let $p(x)$ be a cubic polynomial such that $p(-1) = p(2) = 3$ and $p(1) = p(-1/2) = 0$.

What are the roots of $p(x)$?

a) $\left\{1, -\frac{1}{2}, 4\right\}$ b) $\left\{-1, \frac{1}{2}, -4\right\}$ c) $\left\{1, 1, -\frac{1}{2}\right\}$ d) $\left\{1, -\frac{1}{2}, 2\right\}$ e) $\left\{-1, 1, -\frac{1}{2}\right\}$

45. If $x = \sin 50°$ then

a) $x = \cos 50°$ b) $x = \cos 40°$ c) $x = \cos 30°$ d) $x = \tan 40°$ e) $x = \tan 50°$

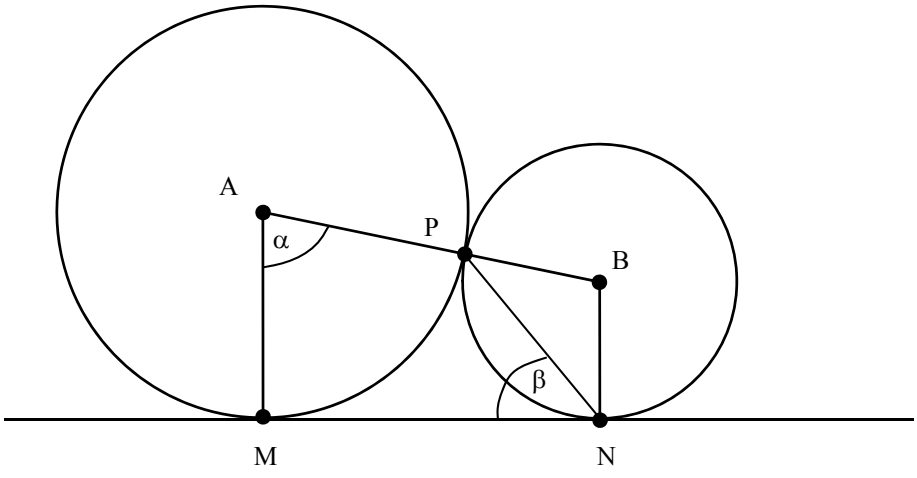
46. Evaluate $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+...}}}}$

a) 2 b) $\sqrt{3}$ c) $\sqrt{2}$ d) 1 e) None listed.

47. For any positive integer n, suppose that α is a solution to the equation $x^{2n} - x^{2n-1} + x^{2n-2} - ... + x^2 - x + 1 = 0$. Evaluate α^{2n+1}.

a) α b) α^2 c) 1 d) -1 e) $-\alpha$

48. Points *A* and *B* are the centers of two circles, tangent to each other at point *P*. Both circles are tangent to a line and points *M* and *N*, respectively. Consider angles $\alpha = \angle MAP$ and $\beta = \angle MNP$. The angle α is equal to



a) $\alpha = 180° - 2\beta$ b) $\alpha = 360° - \beta$ c) $\alpha = \beta$ d) $\alpha = 30° + \beta$ e) $\alpha = 60°$

49. Suppose $\frac{x}{y+1}=1$. Evaluate $\frac{x+1}{y+2}$.

a) 1 b) 2 c) $\frac{3}{2}$ d) $\frac{2}{3}$ e) None listed.

50. Suppose $f(x)=x^3$. Find a real value of a such that $f(f(f(a)))=27a$.

a) -3 b) -1 c) 0 d) 1 e) None listed.

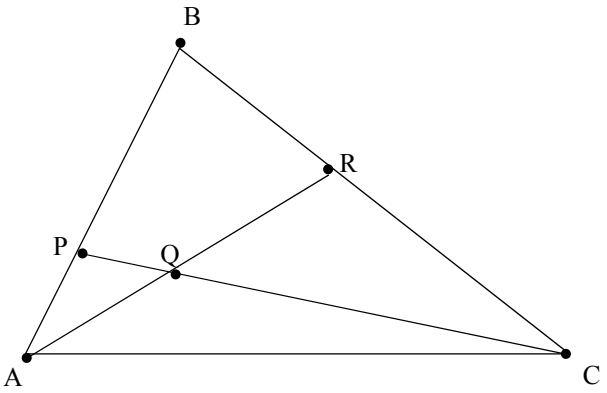
51. How many solutions satisfying the equation $x^x = x$ exist?

a) 0 b) 1 c) 2 d) 3 e) None listed.

52. Suppose p and q are distinct positive integers. Which of the following must be a solution of $(x-p)(x-q)=0$?

a) $x=p-q$ b) $x=q-p$ c) $x=2p$ d) $x=p$ e) None listed.

53. In the given figure, suppose $AP = 2$ cm, $PB = 4$ cm, $BR = 3$ cm, and $RC = 5$ cm. The ratio of RQ to AQ is



a) 1:1 b) 6:5 c) 5:4 d) 4:3 e) None listed.

54. A box contains 6 blue balls, 4 white balls, and 3 red balls. One ball is drawn at random. Its color is noted and the ball is set aside. A second ball is drawn at random. What is the probability that the second ball has the same color as the first?

a) 1/3 b) 4/13 c) 1/2 d) 5/12 e) 3/4

55. Find a real number x satisfying simultaneously $x + y = 1$ and $x^2 - y^2 = 3$.

a) 0 b) 1 c) 2 d) -1 e) None listed.

56. Assume three real numbers $x, y,$ and z satisfy $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 1$. What is the maximal value of $x + y + z$?

a) -2 b) -1 c) 0 d) 1 e) 2

57. What is the value of $1 + \dfrac{2}{3 + \dfrac{4}{5 + \dfrac{6}{7}}}$?

a) 77/50 b) 232/151 c) 233/151 d) 8/5 e) 23/15

58. What is the base 12 representation of 1/9?

a) $.1\bar{1}_{12}$ b) $.1_{12}$ c) $.\overline{13}_{12}$ d) $.14_{12}$ e) $.15_{12}$

59. Suppose α is root of $x^4 + x^2 - 1 = 0$. What is the value of $\alpha^6 + 2\alpha^4$?

a) -1 b) 2 c) 0 d) 1 e) Can't be determined

60. The largest circle that fits inside an equilateral triangle T has area π. Each side of T has length

a) $\sqrt{3}$ b) $2\sqrt{3}$ c) $\pi\sqrt{3}$ d) $3\sqrt{3}$ e) $\frac{3\sqrt{3}}{2}$

61. If the letters A through F are the vertices of a regular hexagon listed in clockwise order, consider the triangle ACE. What is the ratio of the area of the triangle to the area of the hexagon?

a) 1:2 b) 1:3 c) 2:3 d) $1:\sqrt{2}$ e) $1:\sqrt{3}$

62. What is the lateral surface area of the portion of the cylinder $x^2 + y^2 = 9$ that lies on or above the *xy*-plane and on or below the plane $x + 2z = 8$?

a) 15π b) 18π c) 21π d) 24π e) 27π

63. The roots of $120x^4 - 46x^3 - 19x^2 + Ax + 1 = 0$ are $\left\{-\frac{1}{4}, -\frac{1}{5}, \frac{1}{2}, \frac{1}{3}\right\}$. What are the roots of $x^4 + Ax^3 - 19x^2 - 46x + 120 = 0$?

a) $\left\{\frac{1}{4}, \frac{1}{5}, \frac{-1}{2}, \frac{-1}{3}\right\}$ b) {4,5,2,3} c) {–4,–5,2,3} d) $\left\{\frac{-1}{4}, \frac{-1}{5}, \frac{1}{2}, \frac{1}{3}\right\}$ e) None listed.

64. Given the set {1, 2, 3, 4, 5, 6}. All of the subsets are written out. The sums of the elements in each subset is computed and then all of these sums are totaled for a final sum. Which of the following best describes this final sum?

a) Less than 500.
b) Between 500 and 700.
c) Between 700 and 900.
d) Between 900 and 1000.
e) Over 1000.

65. Compute $\frac{1}{2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\ldots+\frac{1}{2017\cdot 2018}$.

a) $\frac{2017}{2018}$ b) $\frac{2018}{2017}$ c) $\frac{2017}{2016}$ d) 1 e) None listed.

66. Suppose that two real numbers x and y satisfy $x^2+y^2=1$. What is the maximum value xy?

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$ e) None listed.

67. What is the smallest integer which DOES NOT solve $x^2+x-6<0$?

a) 2 b) 4 c) -3 d) 3 e) There is no smallest integer which does not solve this.