

1. What is the minimum value of $y = \frac{1}{2}x^2 + 4x + 3$?

- a) -4 b) -5 c) 3 d) 4 e) None listed.

2. The sum of the first n terms of an arithmetic sequence is $2n^2 + 5n$. The third term of this sequence is

- a) 17 b) 18 c) 15 d) 33 e) None listed.

3. The simultaneous equations

$$x - 3y = 4$$

$$2x - 3y = 5$$

have the solution

- a) $x = 9, y = \frac{5}{3}$ b) $x = -3, y = \frac{11}{3}$ c) $x = 1, y = -1$ d) $x = 3, y = -\frac{1}{3}$ e) None listed.

4. In a two-child family, it is known that at least one child is a boy. What is the probability that the other child is a girl?

- a) $1/2$ b) $1/3$ c) $2/3$ d) $3/4$ e) None listed.

5. A certain wanderer traveled the distance of 112 km by walking the same number of kilometers every day. If he could afford for this travel 3 more days, he could travel every day 12 km less. How many km a day did he travel? Find your answer in one of these groups of numbers.

- a) $7, 10, 22$ b) $8, 13, 3$ c) $1, 17, 28$ d) $5, 15, 25$ e) None listed.

6. The area of a right angle is 60 and one of the legs is shorter than the other by 7. What is the length of the hypotenuse?

- a) 1 or 55 b) 17 or 11 c) 12 or 24 d) 18 e) None listed.

7. Suppose $\frac{\sin \varphi}{\cos \varphi} + \frac{\cos \varphi}{\sin \varphi} = 2$. Find the value of $\sin \varphi \cdot \cos \varphi$.

- a) $7/3$ b) $-7/3$ c) $1/2$ d) $3/7$ e) None listed.

8. Benny eats a box of cereal in 14 days. He eats the same size box of cereal with his younger brother Nathan in 10 days. How many days will it take Nathan to finish the box of cereal alone?

- a) 20 b) 25 c) 30 d) 35 e) 40

9. The radius of the circle given by $x^2 - 6x + y^2 + 4y = 12$ is

- a) 5 b) 6 c) 7 d) 8 e) 36

10. Some hikers start on a walk at 9 a.m. and return at 2 p.m. One quarter of the distance walked is uphill, one half is level, and one quarter is downhill. If their speed is 4 miles per hour on level land, 2 miles per hour uphill, and 6 miles per hour downhill, approximately how far did they walk?

- a) 16.4 miles b) 17.1 miles c) 18.9 miles d) 20.0 miles e) 21.2 miles

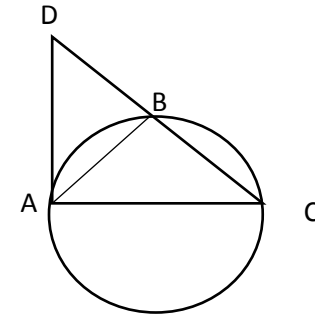
11. In a box there are red and blue balls. If you select a handful of them with eyes closed, you have to grab at least 5 of them to make sure at least one of them is red and you have to grab at least 10 of them to make sure both colors appear among the balls selected. How many balls are there in the box?

- a) 10 b) 11 c) 12 d) 13 e) 14

12. Let x and y be positive real numbers such that $2\log(x-2y) = \log x + \log y$. Find the $\frac{x}{y}$.

- a) 4 b) 5 c) $10^{4\log y}$ d) $(x+2y)^2$ e) None listed.

13. Triangle ABC is inscribed in a circle and $\overline{AB} = 1\text{ cm}$. The tangent to the circle at A meets the second line through B and C at a point D . If B is the midpoint of the segment CD , find the length of the segment AC .



- a) 1 cm b) $\frac{\sqrt{3}}{2}\text{ cm}$ c) $\sqrt{2}\text{ cm}$ d) $\sqrt{6}$ e) None listed.

14. How many integers satisfy the equation $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$?

- a) 2 b) 3 c) 4 d) 5 e) None listed.

15. If $A = 20^\circ$, $B = 25^\circ$, then the value of $(1 + \tan A)(1 + \tan B)$ is

- a) $\sqrt{3}$ b) 2 c) $1 + \sqrt{2}$ d) 4 e) None listed.

16. An athlete covers three consecutive miles by swimming the first, running the second and cycling the third. He runs twice as fast as he swims and cycles one and a half times as fast as he runs. He takes ten minutes longer than he would do if he cycled the whole three miles. How many minutes does he take?
- a) 16 b) 22 c) 30 d) 46 e) 70
17. If $a+b=1$ and $a^2+b^2=7$, then $a^4+b^4=$
- a) 1 b) 8 c) 13 d) 31 e) None listed.
18. The hypotenuse of a right triangle is longer than one of the sides by 1 inch and longer than the other one by 32 inches. Find the lengths of the sides of the triangle. The hypotenuse has length
- a) 41 b) 12 c) 17 d) 100 e) None listed.
19. The angle φ is acute and $\tan \varphi = 4/3$. Find $\sin \varphi + \cos \varphi$.
- a) $7/5$ b) $\sqrt{3/4}$ c) $2/3$ d) $\sqrt{3/2}$ e) None listed.
20. Jim has only nickels and quarters in his pocket. He has 11 more nickels than quarters, and has a total of \$2.65. How many coins does he have in his pocket?
- a) 22 b) 25 c) 18 d) 31 e) None listed.
21. Which term of the arithmetic sequence 2,5,8,... is equal to 227?
- a) 70th Term b) 76th Term c) 80th Term d) 86th Term e) None listed.

22. The inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ is

a) $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

c) $\begin{bmatrix} -1 & -2 \\ -2 & -5 \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{2} & 1 \\ 1 & \frac{1}{5} \end{bmatrix}$

e) None listed.

23. What is the next time after 4:00 p.m. that the hands of the clock will next form a 120° angle?

a) 4:43 $\frac{7}{11}$ p.m. b) 4:43 $\frac{5}{11}$ p.m. c) 4:43 $\frac{2}{11}$ p.m. d) 4:43 $\frac{3}{11}$ p.m. e) None listed.

24. What is the base 4 representation of the base 8 number 713241_8 ?

a) 102221123 b) 321122201 c) 313212211 d) 12022102 e) None listed.

25. How many of the following statements are true?

- I) The degree of 0 is 0.
- II) The degree of $x^3 + 4x^2$ is 5.
- III) The degree of $x^3(4x)^2$ is 5.
- IV) The degree of $(2x+3)^5$ is 5.

a) 0

b) 1

c) 2

d) 3

e) 4

26. Suppose a football conference has 10 teams and each team plays each other team exactly once during the football season. How many conference games are played?

- a) 55 b) 90 c) 35 d) 80 e) 45

27. Let x and y be real numbers. Suppose $\ln(x^2 y^2) = 2\ln x + 2\ln y$.

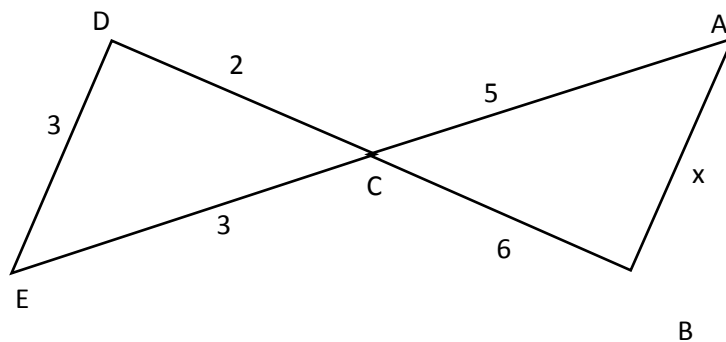
Then

- a) x and y are any real numbers b) $x = y$ c) x and y are positive real numbers d) x and y are nonzero real numbers e) there are no such x and y

28. Dinky Duke and Big Jake are in the lawn mowing business. Jake can mow a large yard in 4 hours while Duke can mow it in 6 hours. If Jake starts a large yard and mows for one hour then quits, how long will Duke take to finish the yard by himself?

- a) 2.4 hrs. b) 3.5 hrs. c) 5 hrs. d) 4.5 hrs. e) None listed.

29. Find the length of AB.



- a) 4 b) 6 c) 5 d) $\sqrt{41}$ e) None listed.

30. The solutions of the equation $2x^2 - 3x - 5 = 0$ are
- a) -1 and 2.5
 - b) -2 and 5
 - c) -1.19 and 4.19 (to 2 d.p.)
 - d) -0.60 and 2.10 (to 2 d.p.)
 - e) None listed.
31. Numbers $x, y, 19$ make an arithmetic sequence (in the given order). Moreover we know that $x + y = 8$. Find x and y . Next, locate the value of the difference $y - x$ in one of the following groups:
- a) 10, 23, 66
 - b) 0, 1, 2, 3
 - c) $-1, -2, -5$
 - d) 11, 12, 13
 - e) None listed.
32. What is the length of the interval of solutions to the inequality $1 \leq 3 - 4x \leq 11$?
- a) 1.75
 - b) 2.00
 - c) 2.25
 - d) 2.50
 - e) 3.25
33. Suppose a, b and c are real numbers for which $a/b > 1$ and $a/c < -1$. Which of the following must be correct?
- a) $a + b - c > 0$
 - b) $a > b$
 - c) $(a - c)(b - c) > 0$
 - d) $a - b + c > 0$
 - e) $abc > 0$
34. A quadrilateral $ABCD$ has vertices with coordinates A (0, 0), B (6, 0), C (5, 4), D (3, 6). What is its area?
- a) 18
 - b) 19
 - c) 20
 - d) 21
 - e) 22

35. The solutions of the equation $x^2 + px + q = 0$ are cubes of the solutions of the equation $x^2 + mx + n = 0$. Which of the following is true?

- a) $p = m^2 + 3mn$ b) $p = m^3 - 3mn$ c) $p = 3mn - m^3$ d) $p + q = m^3$ e) None listed.

36. In $\triangle ABC$, we have $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then $\angle C =$

- a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{3\pi}{4}$ e) None listed.

37. Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on the top of them. What is the distance from the plane to the top of the larger sphere?

- a) $3 + \frac{\sqrt{30}}{2}$ b) $3 + \frac{\sqrt{69}}{3}$ c) $3 + \frac{\sqrt{123}}{4}$ d) $3 + 2\sqrt{2}$ e) None listed.

38. If a, b, c and d are positive integers such that $a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}} = \frac{1990}{991}$,

find the value of d

- a) 12 b) 5 c) 2 d) 1 e) 7

39. Solve the equation $x^3 - 7x^2 + 2x - 14 = 0$. Then find a solution among these groups of numbers

- a) 1, 17, -27, 35 b) 2, 19, -7, 0 c) 17, 7, 335, 211 d) -2, 3, 75, 755 e) None listed.

40. You bought a big cake for a party and expect 10 or 11 people to come. What is the minimal number of pieces (perhaps of different sizes) you need to divide the cake evenly if exactly 10 guests attend or exactly 11 guests attend?

- a) 11 b) 20 c) 30 d) 55 e) 110

41. Cal and Bill use their calculators to calculate $\cos x$. Cal enters x as x degrees while Bill enters x radians. Cal gets an answer y while Bill gets the value $-y$. Let x be the smallest positive value for which this happens. Then $x =$

- a) 3 b) $\pi - 3$ c) $180 - \pi$ d) $\frac{180\pi}{180 + \pi}$ e) None listed.

42. A bug lives on a corner of a cube and is allowed to travel only on edges of the cube. In how many ways can the bug visit each of the other corners once and only once, returning to its home corner only at the end of the trip?

- a) 6 b) 8 c) 12 d) 24 e) None listed.

43. You have 200 coins. You give them to your friends in such a way that each friend gets at least one coin and no friends get the same number coins. What is the largest number of friends that you could have?

- a) 16 b) 17 c) 19 d) 20 e) None listed.

44. What is the area (in square units) of the region of the first quadrant defined by $18 \leq x + y \leq 20$?

- a) 36 b) 38 c) 40 d) 42 e) 44

45. How many positive two-digit integers have an odd number of positive divisors?

- a) 3 b) 4 c) 5 d) 6 e) 7

46. If x is positive, what is the least value of $x + \frac{9}{x}$?

- a) 1 b) 2 c) 3 d) 4 e) 6

47. Let A be the area of a triangle with sides 5, 5, and 8. Let B denote the area of a triangle with sides 5, 5, and 6. Which of the following is true?

- a) $A < B < 12$ b) $B < A < 12$ c) $A = B$ d) $12 < A < B$ e) $12 < B < A$

48. Let N denote the two-digit number whose cube root is the square root of the sum of its digits. How many positive divisors does N have?

- a) 2 b) 3 c) 4 d) 5 e) 6

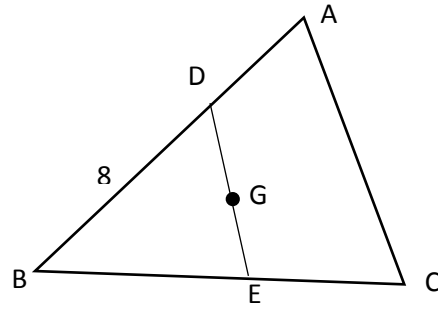
49. Determine the complex number z that satisfies the equation $z + 3\bar{z} = 5 - 6i$, where \bar{z} is the complex conjugate of z .

- a) $\frac{4}{5} + 3i$ b) $-\frac{5}{4} + 3i$ c) $3 + \frac{5}{4}i$ d) $3 - \frac{5}{4}i$ e) None listed.

50. How many zeroes terminate $60!$ in base 7?

- a) 8 b) 9 c) 5 d) 6 e) None listed.

51. Given $\triangle ABC$, with $\overline{DE} \parallel \overline{AC}$, G is the centroid. If the area of $\triangle BDE$ is 40, what is the area of quadrilateral $ADEC$?



- a) 20 b) 40 c) 50 d) 60 e) None listed.

52. If θ is an acute angle, for what value of θ is $\cos(\theta + 6) = \sin(2\theta + 3)^\circ$.

- a) 3° b) 27° c) 39° d) 45° e) None listed.

53. Suppose an amount of \$100,000 is invested at the beginning of the year and withdrawals of \$10,000 are made at the end of the year for 10 years. If the annual interest rate is $i = .10 = 10\%$, let X be the amount of money in the fund at the end of the 11th year after the withdrawal. Find X .

- a) $X = 0$ b) $X = 100000$ c) $X = 110000$ d) $X = 90000$ e) $X = 10000$

54. If x is a positive number such that $x + \frac{1}{x} = 5$, what is the value of $x^3 + \frac{1}{x^3}$?

- a) 52 b) 110 c) 72 d) 68 e) 4.53

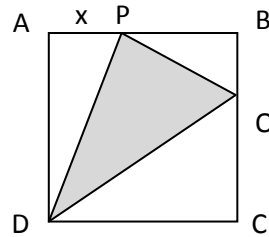
55. Convert 5.13_6 to base 10.

- a) 5.2 b) $5.\bar{2}$ c) 5.1 d) $5.\bar{1}$ e) 5.25

56. Companies whose stocks are listed on the Chicago Stock Exchange have their company name represented by 2 or 3 letters (repetition of letters is allowed.) What is the total number of companies that can be listed on the Chicago Stock Exchange?

- a) 18522 b) 17576 c) 17276 d) 18252 e) None listed.

57. A square $ABCD$ has sides of unit length. Points P and Q are chosen so that $|QC| = 2|AP|$. Find the value of $x = |AP|$ for which the area of the triangle (DPQ) is the least possible.



- a) 1/2 b) 1/3 c) 1/4 d) 2/5 e) None listed.

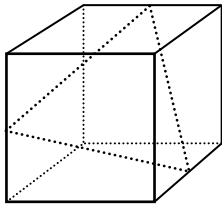
58. Let $\delta_0 = 1 - i$ and $\delta_n = \frac{\delta_{n-1} + i}{\delta_{n-1} - i}$ for all positive integer numbers n . Then $\delta_{2012} =$

- a) $1 - i$ b) $2 - i$ c) $-2 + i$ d) $\frac{1 - 2i}{5}$ e) None listed.

59. Quadrilateral $ABCD$ with the sides $AB = 20$, $BC = 7$, $CD = 24$ and $DA = 15$ has right angles at A and C . What is the area of $ABCD$?

- a) 154 b) 186 c) 200 d) 234 e) 286

60. Find the area of the triangle the vertices of which lie in the middle points of a unit cube (edges of length = 1).



- a) $\frac{3\sqrt{3}}{8}$ b) 2 c) $2\sqrt{3}$ d) $\frac{5\sqrt{5}}{2}$ e) None listed.

61. If $f(x) = \frac{3x-2}{2x+3}$, what is $f^{-1}(x)$?

- a) $\frac{3x+2}{3-2x}$ b) $\frac{2x+3}{2-3x}$ c) $\frac{3x+2}{3+2x}$ d) $\frac{2x-3}{3x+2}$ e) $\frac{2x+3}{3x+2}$

62. Two positive numbers that differ by 3 are multiplied together. The product is added to the sum of the two numbers. The result is 129. What is the smaller of the two numbers?

- a) 6 b) 2 c) 5 d) 9 e) 18

63. Compute $\sqrt{7+\sqrt{40}} + \sqrt{7-\sqrt{40}}$.

- a) $\sqrt{10}$ b) $\sqrt{15}$ c) 6 d) 7 e) $\sqrt{20}$

64. Let x and y be real numbers such that $x + y = 12$. Suppose that $\frac{x}{y} + \frac{y}{x} = \frac{34}{15}$. What is xy ?

- a) 42 b) 135 c) 6.8 d) 33.75 e) Cannot determine.

65. Compute $1 - 2 + 3 - 4 + 5 - 6 + \dots + 99 - 100 + 101$.

- a) -52 b) -50 c) 50 d) 51 e) 52

66. Miyuki has an average of 88 after taking 5 tests. What should she average on the next two tests to average 90 overall?

- a) 95 b) 89 c) 93 d) 99 e) 52

67. Suppose that the average of a , b and c is 27 and the average of b and c and 13 is $d/3$. What is the average of a and d ?

- a) 22 b) 25 c) 33 d) 37 e) 47