

1. Suppose $\frac{x+1}{Y+1} = 1$, what is the value of $\frac{x+3}{Y+3}$?
- a) -1 b) 1 c) $1/3$ d) $4/3$ e) None listed.
2. Which of the following sets cannot represent the lengths of the sides of a triangle?
- a) $\{10, 6, 8\}$ b) $\{10, 6, 3\}$ c) $\{10, 6, 5\}$ d) $\{10, 6, 7\}$ e) All can represent the lengths of the sides of a triangle.
3. The numbers (a, b, c) represent in order an arithmetic progression whose sum is 30. What is the value of b ?
- a) 6 b) 10 c) 15 d) 18 e) 20
4. When the decimal 0.16_{16} (which is in base 16) is converted to a base 10 fraction, the result is
- a) 1 b) $1\ 3/8$ c) $11/64$ d) $11/128$ e) None listed.
5. The equation of a line perpendicular to the line $y = \frac{-1}{3}x + 7$ is
- a) $y - 3x = 2$ b) $3y - x = 5$ c) $2y - 3x = 7$ d) $3y + x = 2$ e) $y = -3x + 7$
6. If all the dimensions of a cuboid are increased by 10%, then the increase in the volume of the cuboid is
- a) $3(1.1) \times 100\%$
b) $3 \times (1.1 - 1) \times 100\%$
c) 31%
d) 33.1%
e) 33%

7. A pupil's marks were wrongly entered as 83 instead of 63. Due to this mistake, the average for the class was increased by one half of a percentage point. The number of pupils in the class is:
- a) 10 b) 20 c) 35 d) 40 e) 73
8. If six more than half of a number is increased by four times one fourth of the number, the result is the number doubled. What is the number?
- a) 4 b) 8 c) 12 d) 24 e) None listed.
9. If the average of the arithmetic and geometric means of two numbers is two, what is the average of the square roots of the two numbers?
- a) $\frac{1}{2}$ b) 1 c) 3 d) $\frac{\sqrt{2}+3}{2}$ e) None listed.
10. The sum of the squares of three consecutive positive integers is 149. Then the largest of these numbers is
- a) 8 b) 9 c) 7 d) 10 e) 11
11. In the sequence of numbers $X, 8, 4, Y$, which ordered pair makes the sequence arithmetic?
- a) (10, 2) b) (12, 0) c) (8, 4) d) (16, 2) e) None listed.
12. Find the sum of all x which solve $|2x - 3| = 21$.
- a) 12 b) -12 c) -3 d) 3 e) None listed.
13. The base of an isosceles triangle is 6 inches long and the sides are each 5 inches long. What is the length of the altitude to the base?
- a) 3 b) 4 c) 5 d) $\sqrt{34}$ e) None listed.

14. In the sequence of numbers $X, 8, 4, Y$, which ordered pair makes the sequence geometric?

- a) (10, 2) b) (12, 0) c) (8, 4) d) (16, 2) e) None listed.

15. The least common multiple of $3, x^2; x^2 - 1; (x + 1)^2$ is

- a) $3x^2(x + 1)^2(x - 1)^2$
b) $3x^2(x + 1)^2(x^2 - 1)$
c) $3x^2(x^2 - 1)(x + 1)$
d) $3x^2(x + 1)(x - 1)$
e) $3x^2(x - 1)^2(x + 1)$

16. In how many ways can 105 be written as the difference of squares of positive integers?

- a) 0 b) 2 c) 4 d) 6 e) 8

17. How many 3 digit numbers which are divisible by five can be formed using the digits 2, 3, 5, 6, 7, 9 if none of its digits are repeated?

- a) 5 b) 10 c) 15 d) 20 e) 25

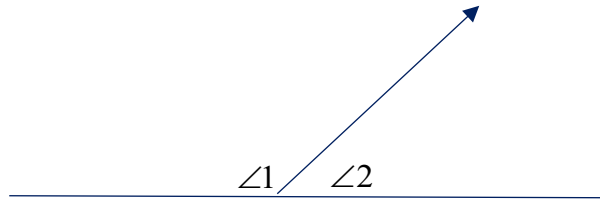
18. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn are blue?

- a) $\frac{10}{21}$ b) $\frac{11}{21}$ c) $\frac{2}{7}$ d) $\frac{5}{7}$ e) $\frac{5}{21}$

19. A sum of \$1360 has been divided among persons A , B , and C . Person A gets $\frac{2}{3}$ of what person B gets, and B receives $\frac{1}{4}$ of what person C gets. Person B 's share is

- a) \$120 b) \$245 c) \$160 d) \$240 e) \$960

20. Given the figure $m\angle 1 = 8x - 1$ and $m\angle 2 = 2x + 21$, find the measure of angle 1.



- a) 55 degrees
b) 16 degrees
c) 127 degrees
d) 53 degrees
e) 151 degrees

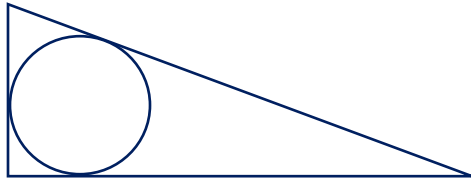
21. Find the coordinates of the midpoint for segment AB if $A(-3, 1)$ and $B(7, 9)$.

- a) (2, 5) b) (5, -4) c) (5, 4) d) (16, 64) e) None listed.

22. Together a bat and a ball cost \$1.10. The bat costs \$1 more than the ball. What is the cost of the ball?

- a) 10 cents b) 15 cents c) 1 dollar d) 25 cents e) None listed.

23. Find the radius of the circle inscribed in the right triangle with two shorter sides 30 and 40. In which of these groups of numbers is it listed?



- a) $\{5, 20, 88\}$
b) $\{9 + 4\sqrt{2}, 15, 16\}$
c) $\{46\sqrt{3}, 17, 18\}$
d) $\{10 + 5\sqrt{2}, 7, 8\}$
e) $\{9, 10, 11\}$
24. Let $G(x)$ be the inverse of $F(x) = 5x + 4$. Then

- a) $G(x) = \frac{x}{5} + 4$
b) $G(x) = \frac{x}{5} - 4$
c) $G(x) = \frac{x + 4}{5}$
d) $G(x) = \frac{x - 4}{5}$
e) $G(x) = -5x - 4$

25. If n is a positive integer, what is the value of $\frac{1 + 3 + 5 + \dots(2n - 1)}{1 + 3 + 5 + \dots(4n - 1)}$?

- a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $2 + n$ d) $n - 3$ e) None listed.

26. If $x - \frac{1}{x} = 2$, what is the value of $\left(x + \frac{1}{x}\right)^2$?

- a) 6 b) 4 c) 3 d) 8 e) 2

27. Which of the following sets can represent the lengths of the sides of a right triangle?

- a) {10, 6, 8} b) {10, 6, 3} c) {10, 6, 5} d) {10, 6, 7} e) All can represent the lengths of the sides of a right triangle.

28. A square has perimeter 16. A second square is formed by connecting the midpoints of the consecutive sides of the original square. A third square is formed by connecting the midpoints of the consecutive sides of the second square. This process is repeated an infinite number of times. What is the sum of the perimeters of all of the squares?

- a) 16 b) $16 + 16\sqrt{2}$ c) $32 + 16\sqrt{2}$ d) 32 e) Infinity.

29. What is the remainder when 12^{12} is divided by 13?

- a) 12 b) 1 c) 3 d) $\frac{1}{13}$ e) None listed.

30. How many solutions are there to the equation $||x| - 2| = |x| + 2$?

- a) None b) 1 c) 2 d) 3 e) 4

31. How many real numbers exist that are solutions to $\sqrt{\sqrt{x}} = -x$?

- a) 1 b) 2 c) 3 d) 4 e) None listed.

32. Evaluate the expression $\sqrt{\sqrt{\sqrt{\dots\sqrt{5}}}}$.

- a) 2 b) $\sqrt{5}$ c) $\sqrt{3}$ d) 1 e) None listed.

33. Compute the sum $1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} \dots$.

- a) 1 b) $\frac{3}{2}$ c) $\sqrt{2}$ d) 2 e) None listed.

34. When in radians find the sum of all positive integers n such that $\frac{1}{4} < \sin\left(\frac{\pi}{n}\right) < \frac{1}{3}$?

- a) 19 b) 21 c) 30 d) 33 e) None listed.

35. Let a and b be the zeroes of $F(x) = -\sqrt{3}(x+3)(x-5)$. Which of the following is true?

- a) $a+b = -8$
b) $a+b = -\sqrt{3}$
c) $a+b = 8$
d) $a+b = 2$
e) $a+b = -2$

36. Suppose p and q are distinct positive integers. Which of the following could be a solution of $(x-p)(x-q) = pq$?

- a) $x = p - q$ b) $x = q - p$ c) $x = 2p$ d) $x = p$ e) None listed.

37. The least perfect square which is divisible by each of 21, 36, and 66 is:

- a) 213444 b) 214344 c) 3214434 d) 231444 e) 234144

38. What is the probability of getting at least two heads if a fair coin is flipped four times?

- a) $\frac{1}{16}$ b) $\frac{2}{16}$ c) $\frac{7}{16}$ d) $\frac{9}{16}$ e) $\frac{11}{16}$

39. Let $f(x)$ be a function satisfying $f(x) + 3f(1-x) = 2x + 7$ for all real x . Then $f(2)$ is

- a) $\frac{1}{2}$ b) 2 c) $\frac{7}{2}$ d) 5 e) None listed.

40. How many 4 letter words, with or without meaning, can be formed out of the letters of the word 'LOGARITHMS' if repetition of letters is not allowed?

- a) 40 b) 400 c) 5040 d) 2520 e) 3025

41. If a circle has area $\frac{125}{\pi}$, what is its diameter?

- a) $\frac{5\sqrt{10}}{\pi}$ b) $\frac{\pi^2}{125}$ c) $\frac{10\sqrt{5}}{\pi}$ d) $\frac{5\sqrt{10}}{\pi^2}$ e) None listed.

42. If two standard dice are rolled and the product of the top two numbers showing is computed, what is the probability this product is even?

- a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{3}{8}$ d) $\frac{5}{16}$ e) $\frac{3}{16}$

43. If $f(x) = x - 5$ and $g(x) = x^2 + 3$, find $(f \circ g)(-2)$.

- a) -6 b) 12 c) 52 d) 2 e) None listed.

44. A rectangle has its length increased by a factor of one-third and its width decreased by a factor of one-third. Then its area is:

- a) Increased by a factor of one-third.
b) Decreased by a factor of one-ninth.
c) Stayed the same.
d) Increased by a factor of eight-thirds.
e) Decreased by a factor of one-third.

45. Let $\cos(x) + \cos(y) = \frac{3}{2}$ and $\sin(x) - \sin(y) = \frac{2}{3}$. What is the value of $\cos(x + y)$?

- a) $\frac{18}{25}$ b) $\frac{15}{14}$ c) $\frac{15}{19}$ d) $\frac{25}{36}$ e) $\frac{25}{72}$

46. In what base b does $(4_b)(12_b) = 103_b$?

- a) 12 b) 7 c) 8 d) 5 e) None listed.

47. If $3^{y-x} = \frac{1}{27}$ and $3^{x+y} = 243$, what is the value of x ?

- a) 0 b) 2 c) 4 d) 6 e) None listed.

48. Find the largest integer K for which $10!$ is divisible by 30^K .

- a) 1 b) 2 c) 3 d) 4 e) None listed.

49. If x and y are simultaneous solutions of the system of equations

$$\begin{array}{l} 2x - y = 3 \\ x + 2y = 7 \end{array} \text{ then } x + y \text{ equals}$$

- a) 3 b) $\frac{10}{3}$ c) $\frac{26}{5}$ d) $\frac{24}{5}$ e) None listed.

50. If θ is an acute angle and $\tan \frac{\theta}{2} = \frac{1}{4}$ then $\sin \theta$ is

- a) $\frac{3}{5}$ b) $\frac{5}{13}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{17}}{17}$ e) None listed.

51. Find the following sum:

$$1 \cdot 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \cdots + \frac{1}{100}(1+2+\cdots+100)$$

- a) 6376 b) 2575 c) 2352.01 d) 3526 e) None listed.

52. If the average income for P and Q is \$5050, the average income for Q and R is \$6250 and for P and R is \$5200, what is the income for P ?

- a) \$3500 b) \$4000 c) \$4050 d) \$5000 e) None listed.

53. A right circular cone has diameter 12 and slant height 10. It is sliced by a plane parallel to its base at the midpoint of its height. What is the total surface area of the resulting frustrum?

- a) 24π b) 45π c) 81π d) 90π e) None listed.

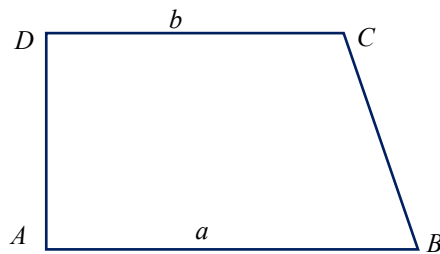
54. The solution set to the inequality $\frac{1-2x}{2} > \frac{2}{3}$ is

- a) $\left(-\infty, -\frac{1}{6}\right)$ b) $\left(-\infty, -\frac{2}{3}\right)$ c) $\left(\frac{1}{6}, \infty\right)$ d) Empty set e) $\left(\frac{2}{3}, \infty\right)$

55. What is the value of $\sqrt{6+\sqrt{11}} + \sqrt{6-\sqrt{11}}$?

- a) $6-\sqrt{2}$ b) $6+\sqrt{2}$ c) $\sqrt{12}$ d) $\sqrt{22}$ e) $\sqrt{21}$

56. Consider the trapezoid $ABCD$ with right angles at A and D and the measure of angle ABC is 60 degrees. If the length of CD is b and the length of AB is a with $a > b$, what is the length of side BC ?



- a) $a-b$ b) $2(a-b)$ c) $a-\sqrt{3}b$ d) $\frac{a-b}{2}$ e) None listed.

57. What positive integer values of x make both $\sqrt{x+56}$ and $\sqrt{x-49}$ positive integers?

- a) { 65, 113, 306, 2752 }
- b) { 65, 113, 305, 2753 }
- c) { 65, 113, 301, 2752 }
- d) { 65, 113, 305, 2751 }
- e) No such integers exist.

58. Solve $5x^2 - 3x + 2 = 0$.

- a) $x = \frac{1}{5}, x = 2$
- b) $x = \frac{2}{3}, x = 1$
- c) $x = \frac{3 \pm \sqrt{31}}{10}$
- d) $x = \frac{3 \pm i\sqrt{31}}{10}$
- e) None listed.

59. If $\log_2 x = 5$ and $\log_4 y = 7$, what is the value of $\log_8 x^2 y$?

- a) 17
- b) 12
- c) $\frac{27}{2}$
- d) 8
- e) None listed.

60. Let $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $A = (\cos \theta)^{\cos \theta}$, $B = (\sin \theta)^{\cos \theta}$ and $C = (\cos \theta)^{\sin \theta}$.

Which of the following is true?

- a) $A < B < C$
- b) $A < C < B$
- c) $B < A < C$
- d) $B < C < A$
- e) $C < A < B$

61. A regular polygon has 135 diagonals. What is the measure of one exterior angle of this polygon?

- a) 360 degrees
- b) 18 degrees
- c) 36 degrees
- d) 20 degrees
- e) None listed.

62. Let a , b , c and d be integers satisfying $a \log 2 + b \log 3 + c \log 5 + d \log 7 = 2019$. Find the value of $a + b + c + d$.

- a) 4038 b) 2019 c) 2020 d) 2018 e) None listed.

63. The polar points $\left(3, \frac{\pi}{2}\right)$ and $\left(7, \frac{3\pi}{2}\right)$ are plotted. What are the polar coordinates of the midpoint of the line segment joining these two points?

- a) $(5, \pi)$ b) $\left(-2, \frac{\pi}{2}\right)$ c) $\left(5, \frac{3\pi}{2}\right)$ d) $(-2, 0)$ e) None listed.

64. Certain problems of computer data use are modeled by the so-called “Dining Philosophers Problem:” Four philosophers sit at a square table, each with a plate of food. Four chopsticks are situated at the corners of the table. At any given time, each philosopher is either thinking or eating. Only a philosopher holding two chopsticks can eat. After eating, a philosopher must put down both chopsticks, to make them available to others. Suppose that each philosopher uses the following algorithm: Until the left chopstick is available, think. When the left chopstick is available, pick it up. Until the right chopstick is available, think. When the right chopstick is available, pick it up. When holding both chopsticks, eat for two minutes. Put down both chopsticks. Repeat. Which of the following is true?

- a) Every philosopher will get a chance to eat during every hour.
b) Every philosopher will get a chance to eat at some point in the future, but not in every hour.
c) Some philosophers will get a chance to eat during every hour, but others will never eat.
d) Some philosophers will get a chance to eat at some point but not in every hour, but others will never eat.
e) All of the philosophers will starve.

65. Find the mean for the data items in the frequency distribution. Give your answer rounded to the nearest tenth.

Score	1	2	3	4	5	6	7	8	9	10
Frequency	3	3	3	4	8	4	5	4	4	3

- a) 5.0 b) 5.5 c) 5.6 d) 6 e) None listed.

66. Which of the following is a zero of the polynomial function $f(x) = x^3 - 3x^2 - 3x + 5$?

- a) -1 b) 0 c) $1 - \sqrt{6}$ d) $1 + \sqrt{6}i$ e) None listed.

67. Five people named A , B , C , D and E are seated in a line and each seating order is equally likely. What is the probability there is exactly one person between A and B ?

- a) $\frac{1}{10}$ b) $\frac{3}{10}$ c) $\frac{3}{5}$ d) $\frac{3}{20}$ e) None listed.