

(18) 1. Find each of the following limits. Give clear reasons for your answers.

(a) $\lim_{x \rightarrow 0} \frac{\tan(2x)}{\arctan(3x)}$

(b) $\lim_{x \rightarrow +\infty} x^2 e^{-3x}$

(c) $\lim_{x \rightarrow 0^+} (\sin(2x))^{3/\ln(4x)}$

(20) 2. Evaluate each of the following indefinite integrals.

(a) $\int (3x + 1)\sin(2x) dx$

(b) $\int \frac{x^2}{\sqrt{1-x^2}} dx$

(20) 3. Evaluate the following definite integrals.

(a)
$$\int_1^2 \frac{(x-2)dx}{(x+2)(x+3)}$$

(b)
$$\int_0^{\pi/4} \tan^{3/2} x \sec^4 x dx$$

- (20) 4. Consider the following improper integrals. Do they converge or diverge? If they converge, evaluate them.

(a) $\int_4^{+\infty} \frac{1}{16+x^2} dx$

(b) $\int_{e^2}^{e^6} \frac{dx}{x \ln x}$

- (40) 5. Determine whether each of the following series converges absolutely, converges conditionally or diverges. List whatever tests you use to justify your argument.

(a)
$$\sum_{n=1}^{+\infty} \frac{6^n (2n+1)^2}{n!}$$

(b)
$$\sum_{n=2}^{+\infty} \left(\frac{\ln(n^2+1)}{3 \ln n} \right)^n$$

(Problem 5 continued)

(c)
$$\sum_{n=2}^{+\infty} \frac{(-1)^n}{n(\ln n)^2}$$

(d)
$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{5n-4}$$

(10) 6. Find the interval and radius of convergence of the power series:

$$\sum_{n=1}^{+\infty} \frac{(2x-3)^n}{\sqrt{n}}$$

(12) 7. Determine the Maclaurin series for the following functions.

(a) $f(x) = \frac{x^3}{3 - x^4}$

(b) $f(x) = \sqrt{x} \sin(\sqrt{x})$

(10) 8. Find the Taylor polynomial of degree 3 for

$$f(x) = \ln(x - 3)$$

about $a = 4$.

(10) 9.

(a) Find the Maclaurin expansion of $\sin(x^2)$.

(b) Approximate

$$\int_0^{.75} \sin(x^2) dx$$

to within .0001.

(10)10. Find an equation of the tangent line to the curve

$$x = 3 + 2 \tan t \quad y = 1 - \sec t$$

at the point $t = \pi / 4$.

(10)11. Eliminate the parameter to find a Cartesian equation for the curve with parametric equation

$$x = 1 - 2 \sin t, y = 2 + 3 \sin t, \quad 0 \leq t \leq \pi.$$

Identify and sketch the curve. Indicate the direction in which the curve is traced as t increases.

(10)12. Find the arclength of the curve

$$x = e^t - t \quad y = 4e^{t/2}$$

over the interval $-5 \leq t \leq 5$.

(10)13. Find the area inside the inner loop of $r = 1 - 2 \cos \theta$.