(18) 1. Find each of the following limits. Give clear reasons for your answers.

(a) $\lim_{x\to 0} \dfrac{\tan(x)-\sin(x)}{x^3}$

(b) $\lim_{x\to +\infty} \left(e^{2x}-x^4\right)^{1/x}$

(c) $\lim_{x\to +\infty} x\sin\left(\dfrac{1}{x}\right)$

(20) 2. Evaluate each of the following indefinite integrals.

(a) $\int \frac{1+x}{x+x^3}\,dx$

(b) $\int \frac{x^2}{\sqrt{x^2+4}}\,dx$

(20) 3. Evaluate the following definite integrals.

(a) $\int_0^1 (x+3)e^{2x}\,dx$

(b) $\int_0^{\pi/3} \cos^{3/2}(x)\sin^3(x)\,dx$

(20) 4. Consider the following improper integrals. Do they converge or diverge? If they converge, evaluate them.

(a) $\int_{-\infty}^{5} \frac{1}{x^2+25}\,dx$

(b) $\int_{3}^{30} \frac{dx}{\sqrt[3]{x-3}}$

(40) 5. Determine whether each of the following series converges absolutely, converges conditionally or diverges. List whatever tests you use to justify your argument.

(a) $\sum_{n=1}^{+\infty} (-1)^n \frac{3n}{3n^2 - 1}$

(b) $\sum_{n=1}^{+\infty} \frac{3^n n!}{(2n)!}$

(Problem 5 continued)

(c) $\displaystyle\sum_{n=1}^{+\infty}\left(\frac{1}{2}\right)^{n-\sqrt{n}}$

(d) $\displaystyle\sum_{n=1}^{+\infty}\frac{(-1)^n \ln n}{n}$

(10) 6. Find the interval and radius of convergence of the power series:

$$\sum_{n=1}^{+\infty} \frac{2^n (3x-2)^n}{\sqrt{n+4}}$$

(Hint: be sure to check the endpoints.)

(12) 7. Determine the Maclaurin series for the following functions.

(a) $f(x) = \dfrac{\arctan(3x)}{x}$

(b) $f(x) = x^4 e^{-x^2}$

(10) 8. Find the Taylor polynomial of degree 3 for

$$f(x) = \frac{2}{3x+1}$$

about $a = -1$.

(10) 9.

(a) Find the Maclaurin expansion of $\sin\left(x^2\right)$.

(b) Approximate

$$\int_0^{.75} \sin(x^2)\,dx$$

to within .0001.

(10)10. Find an equation of the tangent line to the curve

$$x = t^2 - 2 \qquad y = t^3 - 2t + 1$$

at the point $t = 2$.

(10)11. Eliminate the parameter to find a Cartesian equation for the curve with parametric equation

$$x = 1 - 2\sin t, y = 2 + 3\cos t, \quad -\pi \le t \le \pi.$$

Identify and sketch the curve. Indicate the direction in which the curve is traced as t increases.

(10)12. Find the arclength of the curve

$$x = \frac{t^3}{3} + 4 \qquad y = \frac{t^2}{2} - 7$$

over the interval $1 \le t \le 3$.

(10)13. Find the area inside one loop of $r = 2\cos(5\theta)$.