SPRING 2018

(15) 1. Calculate the following limits.

(a) $\lim_{x\to\infty} \frac{x}{(1+x)^4-1}$ $= 0$

(b) $\lim_{x\to 1}(x^2-4)(x^3+5x-1)$ $= (1-4)(1+5-1)$

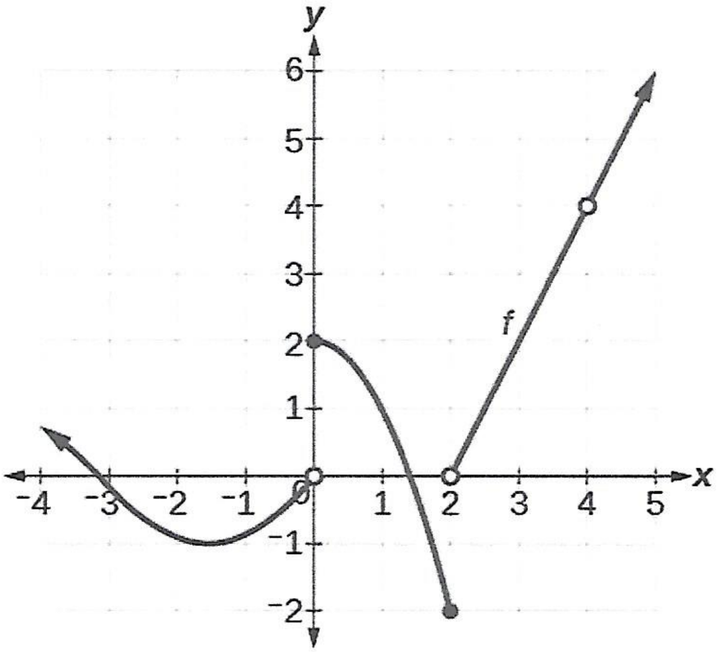
$= (-3)(5)$

$= -15$

(c) $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4}$ $= \lim_{x\to -4} \frac{(x+4)(x+1)}{(x+4)(x-1)}$

$= \lim_{x\to -4} = \frac{-4+1}{-4-1} = \frac{-3}{-5} = \frac{3}{5}$

(12) 2. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a) $\lim_{x\to 0^-} f(x)$ = 0

(c) $\lim_{x\to 2} f(x)$ = DNE

(b) $\lim_{x\to 4^+} f(x)$ = 4

(d) $\lim_{x\to 4} f(x)$ = 4

(28) 3. Compute the derivative of the following functions. **Do not** simplify.

(a) $f(x) = \frac{6\sqrt[3]{x}}{x^2-3}$ $= \frac{6x^{1/3}}{x^2-3}$

$$f'(x) = \frac{(x^2-3)(2x^{-2/3}) - (6x^{1/3})(2x)}{(x^2-3)^2}$$

(b) $f(x) = \ln(x^3 + x) + 2e^x$

$$f'(x) = \frac{3x^2+1}{x^3+x} + 2e^x$$

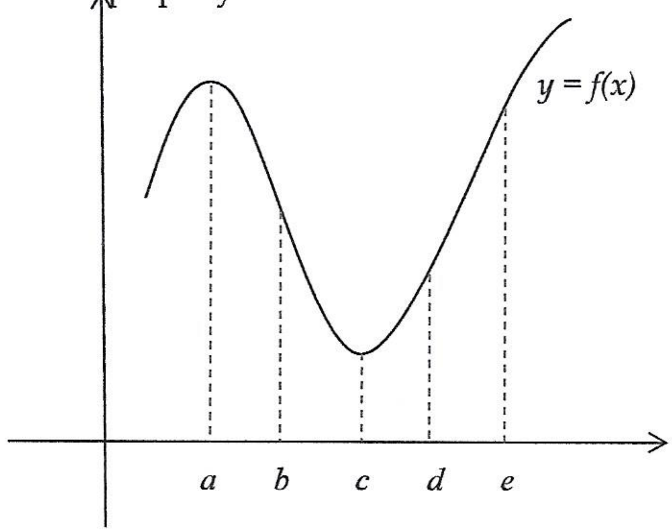
(c) $g(x) = \left(5e^{x^2+1} - x\right)\left(\frac{5}{7}x^2 - 5x + 1\right)$

$$g'(x) = (5e^{x^2+1} - x)(\tfrac{10}{7}x - 5)$$
$$+ (\tfrac{5}{7}x^2 - 5x + 1)(5 \cdot 2xe^{x^2+1} - 1)$$

(d) $h(x) = (3x + 2x^4)^{12} - lnx$

$$h'(x) = 12(3x+2x^4)^{11} \cdot (8x^3+3) - \frac{1}{x}$$

(12) 4. Referring to the given graph, list the labeled value(s) of x at which the derivative has the stated property.



a) $f'(x)$ is positive d, e

b) $f'(x)$ is negative b

c) $f''(x)$ is positive c

d) $f''(x)$ is negative a

(15) 5. Let

$$f(x, y) = x^2y^2 - 5xy^3 + lnx. \text{ Find:}$$

(a) f_x $= 2xy^2 - 5y^3 + \frac{1}{x}$

(b) f_{xx} $= 2y^2 - x^{-2}$

(c) $f_y(e, -1)$ $= 2x^2y - 15xy^2$

$= 2e^2(-1) - 15e(-1)^2$

$= -2e^2 - 15e$

(10) 6. Coffee consumption in the U.S. is greater on a per capita basis than anywhere else in the world. However, due to price fluctuations of coffee beans and worries over the health effects of caffeine, coffee consumption has varied considerably over the years. According to data published in *The Wall Street Journal*, the number of cups $f(x)$ consumed daily per adult is year x (with 1955 corresponding to x=0) is given by

$$f(x) = 0.000144x^3 - 0.00832x^2 + 0.0848x + 2.77.$$

Use $f'(x)$ to predict the rate at which coffee consumption will be increasing or decreasing in the year 2020. Explain your answer.

$f'(x) = 0.000432x^2 - 0.01664x + 0.0848$

$f'(65) = 1.8252 - 1.0816 + 0.0848$

$= 0.8284 > 0$ increasing

(14) 7. Find the consumers' surplus for a printer in an electronics store at the equilibrium price for the given supply and demand functions:

$$D(x) = 25 - 0.004x^2\ ,\ S(x) = 5 + 0.004x^2$$

$-0.004x^2 + 25 = 0.004x^2 + 5$

$-0.008x^2 + 20 = 0$

$-0.008x^2 = -20$

$x^2 = 2500$

$x = 50$

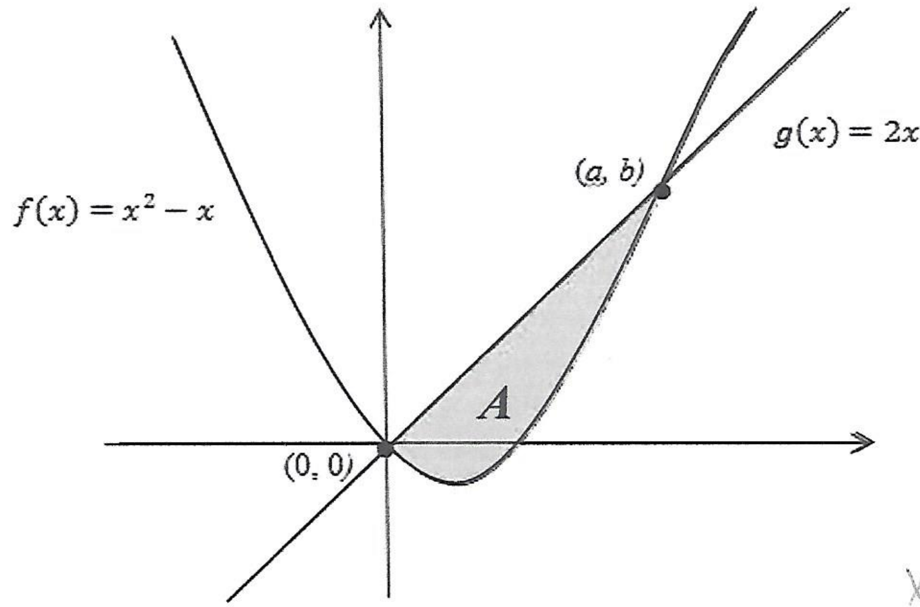
$CS = \int_0^{50} (-0.004x^2 + 25)\,dx - 750 \quad S(50) = 15$

$= -\frac{0.004}{3}x^3 + 25x \Big]_0^{50} - 750$

$\approx -166.667 + 1250 - 750$

$\approx \$1083.33 - 750 \approx \$333.33$

(16) 8. Using the graph of $f(x)$ and $g(x)$ below,



$x^2 - x = 2x$
$x^2 - 3x = 0$
$x(x-3) = 0$
$x = 3, 0$
$g(3) = 6$

a) Find the ordered pair (a, b) above.

$(3, 6)$

b) Find the exact area of region A labeled in the graph.

$$\text{Area} = \int_0^3 [2x - (x^2 - x)]\,dx$$
$$= \int_0^3 (-x^2 + 3x)\,dx = -\frac{1}{3}x^3 + \frac{3}{2}x^2 \Big]_0^3$$
$$= -9 + \frac{27}{2}$$
$$= -\frac{18}{2} + \frac{27}{2}$$
$$= \frac{9}{2}$$

(14) 9. Let $f(x) = \frac{2x-4}{x+2}$

(a) State the equations of the vertical and horizontal asymptotes.

VA: $x=-2$ HA: $y=2$

(b) Give the interval(s) in interval notation over which f is increasing.

$$f'(x) = \frac{(x+2)(2)-(2x-4)(1)}{(x+2)^2}$$

$$= \frac{2x+4-2x+4}{(x+2)^2} = \frac{8}{(x+2)^2} = 8(x+2)^{-2}$$

$+ \quad +$

-2

VA

$(-\infty,-2)$

$(-2,\infty)$

$f'(-3)=8>0 \quad f'(0)=\frac{8}{4}=2>0$

(c) Give the interval(s) in interval notation over which f is concave down.

$$f''(x) = -16(x+2)^{-3} = \frac{-16}{(x+2)^3}$$

$\uparrow \quad \downarrow$

-2

VA

$(-2,\infty)$

$f''(-3)=16>0 \quad f''(0)=\frac{-16}{8}=-2<0$

(28) 10. Compute the following integrals.

(a) $\int \left(8x^{1/3} + \frac{7}{x} - 7\right) dx$

$= 8 \cdot \frac{3}{4} x^{4/3} + 7\ln x - 7x + c$

$= 6x^{4/3} + 7\ln x - 7x + c$

(d) $\int \frac{2x}{1+x^2}\, dx = \int \frac{1}{1+x^2} \cdot 2x\, dx$

$u = 1+x^2$

$du = 2x\, dx$

$= \int \frac{1}{u}\, du = \ln u + c$

$= \ln(1+x^2) + c$

(c) $\int_0^2 x^2\sqrt{x^3+9}\, dx = \int_0^2 u^{1/2} \frac{du}{3} = \frac{1}{3}\int_0^2 u^{1/2}\, du$

$u = x^3 + 9$

$\frac{du}{3} = \frac{3x^2 dx}{3}$

$\frac{du}{3} = x^2 dx$

$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2}$

$= \frac{2}{9}(x^3+9)^{3/2}\Big]_0^2$

$= \frac{2}{9}(17)^{3/2} - \frac{2}{9}(27)$

≈ 9.576

(d) $\int \overset{u}{x}\, \overset{dv}{e^{4x} dx}$ (Use integration by parts.)

$du = dx \quad v = \frac{1}{4}e^{4x}$

$= \frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x} dx$

$= \frac{1}{4}xe^{4} - \frac{1}{16}e^{4x} + c$

(12)11. Find and identify the absolute minimum and maximum values of the function

$$f(x) = 8x^3 - 2x^4$$

on the interval [-2, 2]. Give both coordinates.

$f'(x) = 24x^2 - 8x^3 = 0$

$8x^2(3 - x) = 0$

$x = 0, \cancel{3}$

$f(-2) = 8(-8) - 2(16) = -64 - 32 = -96$ ← min

$f(0) = 0$

$f(2) = 8(8) - 2(16) = 64 - 32 = 32$ ← max

(12)12. A small manufacturing company produces a standard type of surfboard (x) and also a competition-style board (y). Suppose profit is given by

$$P(x, y) = x^3 + y^3 - 6xy.$$

The critical points of $P(x, y)$ are (0, 0), (2, 2). Identify each critical point as a relative minimum, a relative maximum, or a saddle point.

$P_x = 3x^2 - 6y$ $P_y = 3y^2 - 6x$

$P_{xx} = 6x$ $P_{yy} = 6y$

$P_{xy} = -6$

$(0,0) = 6(0) \cdot 6(0) - (-6)^2 = 0 - 36 = -36$ saddle point

$(2,2) = 6(2) \cdot 6(2) - (-6)^2 = 144 - 36 = 108 > 0$

$P_{xx}(2,2) = 12 < 0$

rel. min:

(12)13. Using the Method of Lagrange Multipliers, minimize
$f(x,y) = 3x^2 + 5y^2$ subject to the constraint $g(x,y) = 2x + 3y - 6$.
Give where the minimum occurs as an ordered pair, and then state the minimum value.

$F(x,y,\lambda) = 3x^2 + 5y^2 - \lambda(2x+3y-6)$

$F_x = 6x - 2\lambda = 0 \rightarrow 2\lambda = 6x$
$\lambda = 3x$

$F_y = 10y - 3\lambda = 0$

$F_\lambda = -(2x+3y-6) = 0$

$10y - 3(3x) = 0$

$(-9x + 10y = 0)(-3)$
$(2x + 3y = 6)(10)$

$27x - 30y = 0$
$+\ 20x + 30y = 60$

$47x = 60$

$\boxed{x = \frac{60}{47}}$

$\frac{120}{47} + 3y = 6$

$3y = \frac{282-120}{47}$

$y = \frac{162}{47} \cdot \frac{1}{3} = \boxed{\frac{54}{47}}$

$\boxed{f\left(\frac{60}{47}, \frac{54}{47}\right) \approx 11.489}$