SHOW ALL WORK AND SIMPLIFY ANSWERS (UNLESS OTHERWISE STATED).

1. (12 pts) Calculate the following limits. If the limit does not exist, state so.

a) $\lim\limits_{x \to 1} \frac{x^2+6x-7}{x^2+2x-3}$

b) $\lim\limits_{x \to 2} \frac{\sqrt{x^2+5}}{-x}$

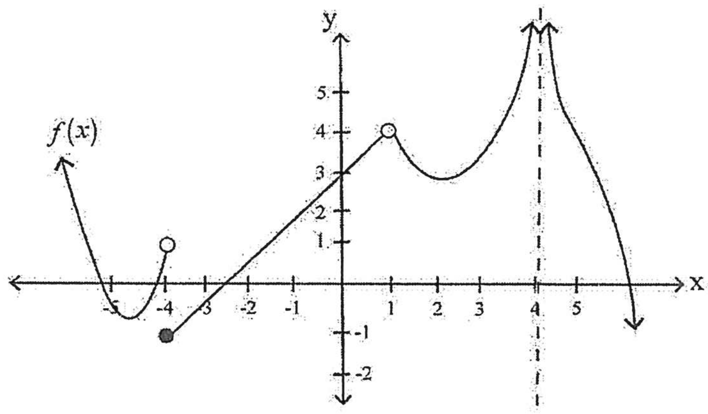
c) $\lim\limits_{x \to \infty} \frac{-x^3+6x^2+3}{5x^5+7x^4-10}$

2. (8 pts) A sewage treatment plant uses a pipeline that extends 1 mile toward the center of a large lake. The concentration of effluent *E(p)* in parts per million, *p* meters from the end of the pipe is given approximately by

$$E(p) = 1000\sqrt{p^2 + 25p}$$

What is the instantaneous rate of change of concentration at 100 meters? Round your answer to the nearest tenth and be sure to use correct units.

3. (15 pts) Problems (a) – (f) refer to the following graph of $y = f(x)$.



a) $\lim\limits_{x\to -4^-} f(x)$

b) $\lim\limits_{x\to -4} f(x)$

c) $\lim\limits_{x\to 1} f(x)$

d) $\lim\limits_{x\to 4^+} f(x)$

e) $\lim\limits_{x\to 0} f(x)$

4. (10 pts) Find the equation of the tangent line to the graph of the function at the indicated value of x.

$$f(x) = (x+2)\ln(1 - x^2 + x^4);\ x = 1$$

5. (21 pts) Compute the derivative of the following functions. DO NOT simplify.

a) $f(x) = 3e^{-6x} + x^{-1/4}$

b) $f(x) = \frac{2\sqrt{x}}{x^2-3x+11}$

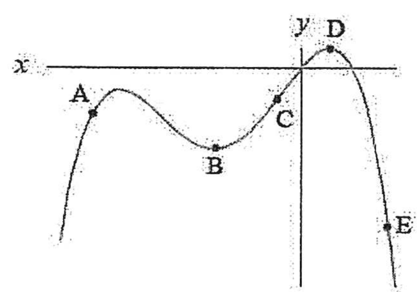
c) $f(x) = (3x^2 - 2x + 3)(4x^2 + 5x - 1)$

6. (10 pts) Find the absolute maximum and/or absolute minimum values for the function

$$R(p) = p^4 - 4p^3 + 5$$

on the interval $[-1, 2]$.

7. (12 pts) Use the graph of $y = f(x)$ below to identify ALL labeled values of x at which the derivative has the stated property:



a) $f'(x) > 0$

b) $f'(x) < 0$

c) $f'(x) = 0$

d) $f''(x) > 0$

8. (10 pts) The rate of change of the income produced by a vending machine is given by

$$I'(t) = 5000e^{0.04t}$$

where t is time in years since the installation of the machine. Find the total income, $I(t)$, produced by the machine during the first 5 years of operation (i.e. from $t=0$ to $t=5$). Round to the nearest cent.

9. (7 pts) Find $f(x)$ such that $f'(x) = 2x - \frac{1}{x^2}$, with the initial condition $f(1) = 5$.

10. (21 pts) Compute the following indefinite integrals.

a) $\int \left(6\sqrt{x}+\frac{4}{x}-17\right)\,dx$

b) $\int x(x^2+16)^6\,dx$

c) $\int \sqrt{x}\,lnx\,dx$ (Use Integration by Parts)

11. (10 pts) Kid First Technology, Inc. has just started producing tablets for children. *D(t)* below is the price parents are willing to pay for *t* tablet(s), and *S(t)* is the price, in dollars, that the company is willing to accept for *t* units.

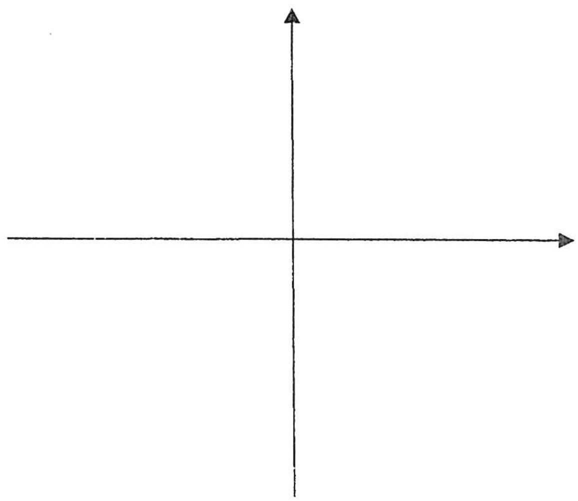
$$S(t) = 200 - 0.02t^2 \qquad D(t) = 100 + t$$

a. Find the coordinates of the equilibrium point.

b. Find the consumer surplus at the equilibrium point.

12. (10 pts) Suppose *f(x)* is a function satisfying the following. Sketch the graph of *f(x)*.

Domain: All real *x*, except *x* = 1;
$f(0) = -2,\ f(2) = 0$;
$f'(x) < 0\ on\ (-\infty, 1)\ and\ (1, \infty)$;
$f''(x) < 0\ \ on\ \ (-\infty, 1)$;
$f''(x) > 0\ \ on\ \ (1, \infty)$;
Vertical asymptote: *x* = 1;
Horizontal asymptote: *y* = -1.



13. (12 pts) Let $G(x, y) = x^2 lny - 3x - 2y + 1$. Find

a) G_x

b) G_{xy}

c) $G_y(-1,4)$

14. (12 pts) Given the function $f(x) = \frac{2x-4}{x+2}$,

a. State the domain of f.

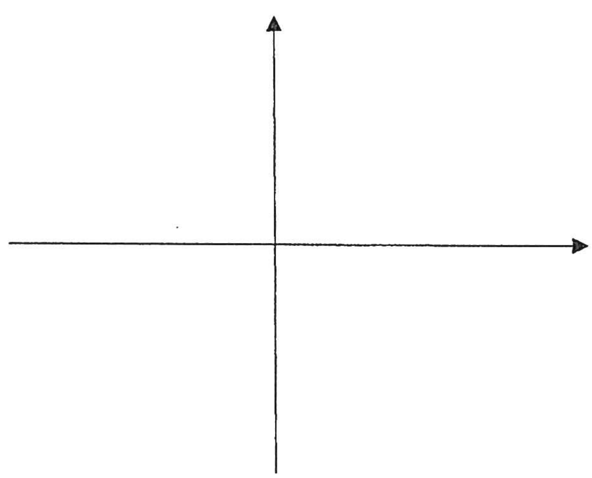
b. Find the vertical asymptote(s).

c. Find the horizontal asymptote(s).

d. Give the interval(s) over which f is increasing. Show work using the First Derivative Test.

e. Give the interval(s) over which f is concave up. Show your work using calculus.

f. Sketch the graph of f on the axes. Be sure to label the asymptotes, x- and y- intercepts. Plot additional points as needed.



15. (10 pts) Given the two functions,

$$f(x) = 5 - x^2 \text{ and } g(x) = 2 - 2x$$

a. Find the ordered pairs where f and g intersect.

b. Sketch a graph of f and g.

c. Find the exact area of the region bounded by the graphs of f and g.

16. (10 pts) Let $f(x) = 2y^3 - 6xy - x^2$.
The critical points of $f(x)$ are $(0, 0)$ and $(9, -3)$. Identify them as a relative minimum, relative maximum, or saddle point.

17. (10 pts) Use Lagrange multipliers to find the maximum value of the function

$$f(x,y) = 25 - x^2 - y^2$$

subject to the constraint

$$2x + y = 10.$$