[24 pts] 1. Find the derivative of each function. Do not simplify.

(a) $f(x) = \dfrac{2}{\sqrt{x}} - 7x^3 - 10$

(b) $f(x) = \dfrac{1 - 2x}{3x^2 + x}$

(c) $f(x) = xe^{5x}$

[12 pts] 2. Find an equation of the line tangent to the curve $y = x \ln x$ at the point (e, e).

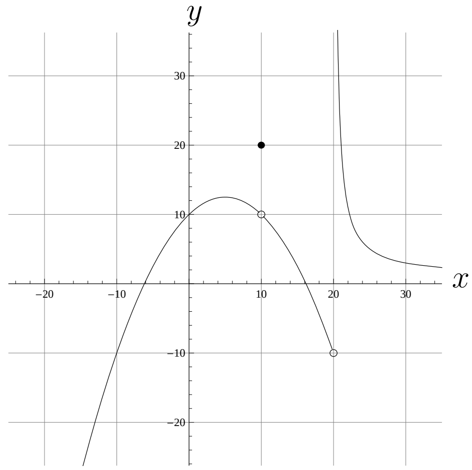
[8 pts] 3. Use the graph of $y = f(x)$ below to estimate each limit if it exists. If the limit does not exist, write "DNE".

(a) $\lim_{x \to 10} f(x) =$ ______

(b) $\lim_{x \to 20^-} f(x) =$ ______

(c) $\lim_{x \to 20^+} f(x) =$ ______

(d) $\lim_{x \to 20} f(x) =$ ______

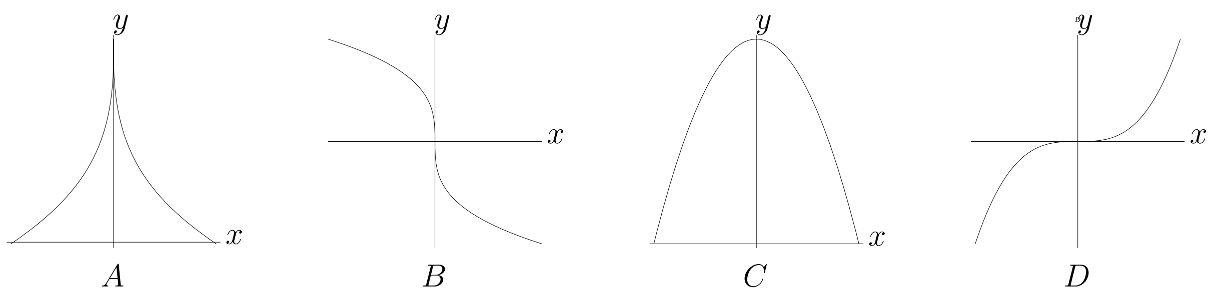


[14 pts] 4. Find each limit, if it exits. If the limit does not exist, write "DNE".

(a) $\lim_{x \to 3} \dfrac{2x^2 - 18}{x^2 + x - 12}$

(b) $\lim_{x \to \infty} \dfrac{4 - x^2}{3x^2 + 9x - 30}$

[12 pts] 5. Match each description below to one of the graphs.



(a) ______ $f'(x) > 0$ and $f''(x) < 0$ on $(-\infty, 0)$, $f'(x) > 0$ and $f''(x) > 0$ on $(0, \infty)$

(b) ______ $f'(x) > 0$ and $f''(x) > 0$ on $(-\infty, 0)$, $f'(x) < 0$ and $f''(x) > 0$ on $(0, \infty)$

(c) ______ $f'(x) > 0$ and $f''(x) < 0$ on $(-\infty, 0)$, $f'(x) < 0$ and $f''(x) < 0$ on $(0, \infty)$

(d) ______ $f'(x) < 0$ and $f''(x) < 0$ on $(-\infty, 0)$, $f'(x) < 0$ and $f''(x) > 0$ on $(0, \infty)$

[14 pts] 6. A company is marketing a new refrigerator. It determines that in order to sell x refrigerators, the price per refrigerator must be

$$p = 200 - 0.2x.$$

It also determines that the total cost of producing x refrigerators is given by

$$C(x) = 1000 + 80x + 0.3x^2.$$

How many refrigerators must the company produce and sell in order to maximize profit? (Recall that the total revenue is $R(x) = px$.)

[16 pts] 7. Let $f(x) = \dfrac{x^3 + 3x^2 - 9x}{20}$.

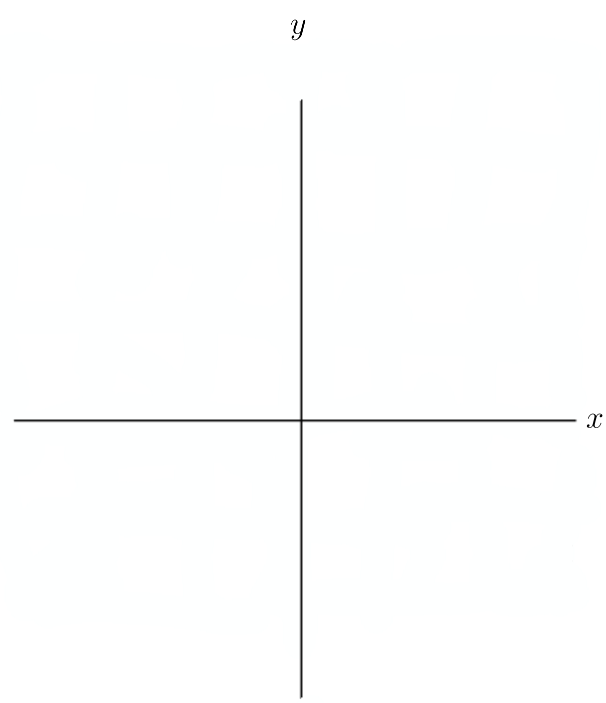
(a) Find the interval(s) on which $f(x)$ is increasing.

(b) Find all points where relative maxima and minima occur.

(c) Find the interval(s) on which $f(x)$ is concave up.

(d) Find all points of inflection.

(e) Sketch on the given axes the graph of $y = f(x)$. Mark the x- and y-intercepts. Plot additional points as needed.



[40 pts] 8. Compute each integral.

(a) $\int \left(x^2 - 5 + \frac{3}{x} \right) dx$

(b) $\int_0^1 xe^{-x^2} dx$

(c) $\int x^2 \sqrt{5 + 2x^3}\ dx$

(d) $\int (x-1) \ln x \, dx$ (Use integration by parts.)

[12 pts] 9. A company determines that its marginal revenue, in dollars, from the sale of x units of a product is given by

$$R'(x) = \frac{2000}{\sqrt[3]{x+1}}.$$

Find the increase in revenue when x increases from 7 to 26.

[14 pts] 10. Let $D(x) = (x-2)^2 + 6$ be the price, in dollars per unit, that consumers are willing to pay for x units of a product. Let $S(x) = x^2 + x$ be the price, in dollars per unit, that producers are willing to accept for x units.

(a) Find the equilibrium point.

(b) Compute the consumer surplus at the equilibrium point, [10 pts] 11. Given $f(x, y) = \frac{1}{2}\ln(x^2 + y^2) + \frac{3y}{x} - 9x$, find:

(a) f_y

(b) f_{yx}

[12 pts] 12. Let

$$f(x, y) = -x^3 - 3y^2 + 6xy + 5.$$

The critical points for this function are $(0, 0)$ and $(2, 2)$. Classify each critical point as a relative maximum, a relative minimum, or a saddle point.

[12 pts] 13. Use the method of Lagrange multipliers to find the minimum value of the function

$$f(x, y) = 2y^2 - 6x^2$$

subject to the constraint

$$2x + y = 4.$$