Math 140   Algebra Review   Name _____

I. Change each to radical form: Recall $x^{m/n} = \sqrt[n]{x^m}$.

   1. $10^{1/2}$ 4. $4a^{1/3}$

   2. $x^{3/2}$ 5. $(4a)^{1/3}$

   3. $x^{2/3}$ 6. $(x+2)^{5/4}$

II. Change to exponent form.

   7. $\sqrt{y}$ 10. $\sqrt{(x+1)^3}$

   8. $\sqrt[3]{y}$ 11. $\sqrt{x^a}$

   9. $\sqrt[3]{y^2}$ 12. $\sqrt[q]{x}$

III. Find the exact value of each. Do not use a calculator. Recall $x^{-m} = \dfrac{1}{x^m}$, $\dfrac{1}{x^{-m}} = x^m$

   13. $16^{1/2}$ 18. $\left(\dfrac{1}{2}\right)^{-3}$

   14. $16^{3/2}$ 19. $\left(\dfrac{4}{9}\right)^{-2}$

   15. $9^{-1/2}$ 20. $\left(\dfrac{4}{9}\right)^{1/2}$

   16. $(-27)^{-1/3}$ 21. 5°

   17. 4^{-2} 22. $2x^\circ$

IV. Express in simplest form. Recall $x^a \cdot x^b = x^{a+b}$, $(x^a)^b = x^{ab}$, $\dfrac{x^a}{x^b} = x^{a-b}$

23. $\left(x^2\right)^5$

27. $\left(x^{1/2}\right)^{1/2}$

31. $\left(x^{-2}\right)^5$

24. $x^2\left(x^5\right)$

28. $x^{1/2} \cdot x^{1/2}$

32. $x^{-2} \cdot x^{-5}$

25. $\dfrac{x^2}{x^5}$

29. $x^2 \cdot x^{1/2}$

33. $\dfrac{x^{-2}}{x^5}$

26. $\dfrac{x^5}{x^2}$

30. $\dfrac{x^2}{\sqrt{x}}$

34. $\dfrac{x^{-2}}{x^{-5}}$

V. Unlike Algebra, calculus does not "OBJECT" to NEGATIVE EXPONENTS. Oftentimes, they are preferred. In algebra, $\dfrac{4}{x^3}$ is the "preferred" form. In calculus, we must change $\dfrac{4}{x^3}$ into $4x^{-3}$ so that we can *differentiate* (or integrate) this expression. Here's a few more examples of this "calculus" form.

$$\frac{2}{x^4} \rightarrow 2x^{-4} \qquad\qquad \frac{1}{(x+1)^2} = (x+1)^{-2}$$

$$\frac{5}{\sqrt{x}} \rightarrow 5x^{-1/2} \qquad\qquad \frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x} = \frac{1}{3}x^{-1} \;\textbf{ or }\; \frac{x^{-1}}{3}$$

Change to "calculus" form.

35. $\dfrac{-2}{x^2}$

37. $\dfrac{5}{(x-2)^3}$

39. $\dfrac{1}{\sqrt[3]{(x-1)^2}}$

36. $\dfrac{1}{\sqrt[3]{x}}$

38. $\dfrac{2}{5x}$

40. $\dfrac{2}{e^x}$

VI. Perform the indicated operation, express in simplest form.

41. $3x^2(2x-5)$

42. $x^{-2}\left(3x^2+x^5\right)$

43. $\sqrt[3]{x}(x-3)$

44. $x\sqrt{x-3}$

45. $\dfrac{x^4+2x^2+8}{x}$

46. $\dfrac{x^4+2x^2+8}{\sqrt[3]{x}}$

Put into calc form

Ex. $\dfrac{2x^2+x+3}{x}=\dfrac{2x^2}{x}+\dfrac{x}{x}+\dfrac{3}{x}$

$=2x+1+3x^{-1}$

47. $(2x-3)(x+5)$

48. $(2x-3)(2x+3)$

49. $(2x-3)^2$

50. $x(2x-3)^2$

51. $(2x-3)^3$

52. $\left(\sqrt{2x-3}\right)^2$

53. $\left(\sqrt{2x}-3\right)^2$

54. $\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)$

55. $3x(x-1)+(x-1)(x-4)$

56. $3x^2-5x-x^2-2x-3$

57. $3x^2-5x-\left(x^2-2x-3\right)$

Compare these two problems. What is the "effect" of the parentheses?

VII. Factor each completely.

58. $2x^2 - 9x$

59. $2x^3 - 10x$

60. $9 - x^2$

61. $2x^3 - 18x$

62. $x^2 - 5x + 6$

63. $x^2 - 5x - 6$

64. $x^2 + x - 6$

65. $2x^2 - 7x + 3$

66. $2x^2 - 5x - 3$

67. $x^3 + 3x^2 - 10x$

68. $xe^x + 2e^x$

69. $x^2 e^{2x} - xe^{2x}$

70. $x^2(x-3)^3 + 4(x-3)^4$

$\left.\begin{array}{l}\\\\\\\\\end{array}\right\}$ Start by common factoring.

71. $x^3 - 3x^2 + 4x - 12$

72. $x^3 - 3x^2 - 4x + 12$

$\left.\begin{array}{l}\\\\\end{array}\right\}$ Factor by grouping.

VIII. Simplify.

73. $\dfrac{2x}{x^2 - 3x}$

74. $\dfrac{x^2 - 3x}{x^2 - 5x + 6}$

75. $\dfrac{x^2 - 3x}{9 - x^2}$

76. $\dfrac{(x-1)2x - x^2(1)}{(x-1)^2}$

Example:
$$x^3 + x^2 - 5x - 5$$
$$= x^2(x+1) - 5(x+1)$$
$$= (x^2 - 5)(x+1)$$

IX. Solve for *x*.

77. $2(3x+4)-(2x-7)=8$

78. $x(2x-3)(x+5)=0$

79. $9x^2-4x=0$

80. $x^3+3x^2-10x=0$

81. $1-\dfrac{5}{x}+\dfrac{6}{x^2}=0$ Start by multiplying by x^2

82. Solve by factoring, then by quadratic formula: $2x^2-5x=3$

83. $x(x-6)=5$ 84. $9x^2-4=0$

85. $.02x^2+.3x-5=0$ Start by multiplying by 100.

86. $\dfrac{x^2-2x}{x^2-9}=0$ When does a fraction $=0$?

X. Sometimes we will have reason to change a fraction into an equivalent form. To do this requires multiplying by a "form of 1". Here are 2 examples along with practice problems.

87. Rationalizing:

Ex. $\dfrac{\sqrt{x}-1}{x-1} = \dfrac{\sqrt{x}-1}{x-1}\left(\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right) = \dfrac{x-1}{(x-1)(\sqrt{x}+1)} = \dfrac{1}{\sqrt{x}+1}$

Practice:

a) $\dfrac{\sqrt{x}-2}{x-4}$

b) $\dfrac{x-9}{\sqrt{x}-3}$

c) $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

88. Replacing powers of x with fractional powers:

Ex. $\dfrac{2x-3}{3x+5} = \dfrac{2x-3}{3x+5}\left(\dfrac{\frac{1}{x}}{\frac{1}{x}}\right) = \dfrac{\frac{2x}{x}-\frac{3}{x}}{\frac{3x}{x}+\frac{5}{x}} = \dfrac{2-\frac{3}{x}}{3+\frac{5}{x}}$

Practice

a) $\dfrac{1+2x}{3x+5}$; multiply by $\dfrac{\frac{1}{x}}{\frac{1}{x}}$

b) $\dfrac{x^2-3}{2x^2+4x}$; multiply by $\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}$

c) $\dfrac{x^2-4x+10}{x^3+6x^2+3x}$; multiply by $\dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}$

1) $\sqrt{10}$ 2) $\sqrt{x^3}$ 3) $\sqrt[3]{x^2}$ 4) $4\sqrt[3]{a}$ 5) $\sqrt[3]{4a}$ 6) $\sqrt[4]{(x+2)^5}$ 7) $y^{\frac{1}{2}}$

8) $y^{\frac{1}{3}}$ 9) $y^{\frac{2}{3}}$ 10) $(x+1)^{\frac{3}{2}}$ 11) $x^{\frac{a}{2}}$ 12) $x^{\frac{1}{a}}$ 13) $\sqrt{16}=4$

14) $(\sqrt{16})^3 = 4^3 = 64$ 15) $(\sqrt{9})^{-1} = 3^{-1} = \dfrac{1}{3}$ 16) $\left(\sqrt[3]{-27}\right)^{-1} = (-3)^{-1} = -\dfrac{1}{3}$

17) $\left(\dfrac{1}{4}\right)^2 = \dfrac{1}{16}$ 18) $2^3 = 8$ 19) $\left(\dfrac{9}{4}\right)^2 = \dfrac{81}{16}$ 20) $\sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$ 21) 1 22) $2(1)=2$

23) x^{10} 27) $x^{\frac{1}{4}}$ 31) x^{-10} or $\dfrac{1}{x^{10}}$ 41) $6x^3 - 15x^2$

24) x^7 28) x 32) x^{-7} or $\dfrac{1}{x^7}$ 42) $3x^0 + x^3 = 3 + x^3$

25) x^{-3} or $\dfrac{1}{x^3}$ 29) $x^{\frac{5}{2}}$ 33) x^{-7} or $\dfrac{1}{x^7}$ 43) $x^{4/3} - 3x^{1/3}$

26) x^3 30) $x^{\frac{3}{2}}$ 34) x^3 44) Can't be multiplied.

35) $-2x^{-2}$ 37) $5(x-2)^{-3}$ 39) $(x-1)^{\frac{-2}{3}}$ 45) $x^3 + 2x + 8x^{-1}$

36) $x^{-\frac{1}{3}}$ 38) $\dfrac{2x^{-1}}{5}$ 40) $2e^{-x}$ 46) $x^{\frac{11}{3}} + 2x^{\frac{5}{3}} + 8x^{-\frac{1}{3}}$

47) $2x^2 + 7x - 15$ 48) $4x^2 - 9$ 49) $4x^2 - 12x + 9$ 50) $x(4x^2 - 12x + 9) = 4x^3 - 12x^2 + 9x$

51) $(2x-3)(4x^2 - 12x + 9) = 8x^3 - 36x^2 + 54x - 27$ 52) $2x - 3$ 53) $2x - 6\sqrt{2x} + 9$

54) $x - 4$ 55) $4x^2 - 8x + 4$ 56) $2x^2 - 7x - 3$ 57) $2x^2 - 3x + 3$

58) $x(2x-9)$ 59) $2x(x^2 - 5)$ 60) $(3-x)(3+x)$ 61) $2x(x^2 - 9) = 2x(x-3)(x+3)$

62) $(x-3)(x-2)$ 63) $(x-6)(x+1)$ 64) $(x+3)(x-2)$ 65) $(2x-1)(x-3)$

66) $(2x+1)(x-3)$ 67) $x(x^2 + 3x - 10) = x(x+5)(x-2)$ 68) $e^x(x+2)$

69) $e^{2x}(x^2 - x)$

$= e^{2x}x(x-1)$

70) $(x-3)^3\left[x^2 + 4(x-3)\right]$

$= (x-3)^3(x^2 + 4x - 12)$

$= (x-3)^3(x+6)(x-2)$

71) $x^2(x-3) + 4(x-3)$

$= (x^2 + 4)(x-3)$

72) $x^2(x-3) - 4(x-3)$

$= (x^2 - 4)(x-3)$

$= (x-2)(x+2)(x-3)$

73) $\dfrac{2x}{x(x-3)} = \dfrac{2}{(x-3)}$

74) $\dfrac{x(x-3)}{(x-3)(x-2)} = \dfrac{x}{x-2}$

75) $\dfrac{x(x-3)}{(3-x)(3+x)} = \dfrac{-x}{3+x}$

Recall $\dfrac{a-b}{b-a} = -1$

76) $\dfrac{2x^2 - 2x - x^2}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$

77) $4x + 15 = 8$

$= x = -\dfrac{7}{4}$

78) $x = 0, \dfrac{3}{2}, -5$

79) $x(9x - 4) = 0$

$\Rightarrow x = 0,\ x = \dfrac{4}{9}$

80) $x(x^2 + 3x - 10) = 0$

$x(x+5)(x-2) = 0$

$x = 0, -5, 2$

81) $x^2\left(1 - \dfrac{5}{x} + \dfrac{6}{x^2}\right) = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0$

$\Rightarrow x = 3,\ x = 2$

82) By factoring: $(2x+1)(x-3) = 0 \Rightarrow x = -\dfrac{1}{2}, x = 3$

By formula: $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$\Rightarrow x = \dfrac{5 \pm 7}{4}$

$\Rightarrow x = \dfrac{12}{4} = 3,\ x = \dfrac{-2}{4} = -\dfrac{1}{2}$

83) $x^2 - 6x - 5 = 0$

$\Rightarrow x = \dfrac{6 \pm \sqrt{56}}{2} \Rightarrow x \approx 6.75, -.75$

When using formula (or factoring) set equation = 0.

84) $9x^2 = 4$

$x^2 = 4$

$x = \pm\sqrt{\dfrac{4}{9}} = \pm\dfrac{2}{3}$

Factoring also works.

85) $2x^2 + 30x - 500 = 0$

$x^2 + 15x - 250 = 0$

$(x+25)(x-10) = 0$

$x = -25,\ 10$

86) A fraction equals 0 when its numerator equals 0. (Dividing by 0 is undefined).

$\dfrac{x^2 - 2x}{x^2 - 9} = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0;\ x = 0,\ 2$

87 a) $\dfrac{\sqrt{x}-2}{x-4}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\dfrac{x-4}{(x-4)(5x+2)}=\dfrac{1}{\sqrt{x}+2}$

   b) $\dfrac{x-9}{\sqrt{x}-3}\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\dfrac{(x-9)(\sqrt{x}+3)}{x-9}=\sqrt{x}+3$

   c) $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\left(\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)=\dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

88 a) $\dfrac{\dfrac{1}{x}+2}{3+\dfrac{5}{x}}$ b) $\dfrac{1-\dfrac{3}{x^2}}{2+\dfrac{4}{x}}$ c) $\dfrac{\dfrac{1}{x}-\dfrac{4}{x^2}+\dfrac{10}{x^3}}{1+\dfrac{6}{x}+\dfrac{3}{x^2}}$