Name: Key Section:

Math 125 Final Exam, Spring 2018

- The following exam has 4 parts, 21 problems, and 8 pages. Please stop and make sure that your exam has all its pages.
- Please raise your hand if you have any questions or need a restroom break.
- When you have completed your exam, raise your hand and the instructor will collect your exam. DO NOT begin packing up until you have turned your exam in.
- ANY use of cell phones or electronics other than an appropriate calculator will result in you receiving a zero on your final exam.
- ANY cheating (cheat sheets, communicating with classmates, etc.) will result in you receiving a zero on your final exam.

Section 1: Quick problems. Show work to receive partial credit. Make sure you simplify fully and round appropriately.

1. [5]Complete the indicated operation and give your answer in scientific notation. Round appropriately as your final step:
   a. $2.1 \times 10^{-3} - 8.1 \times 10^{-4}$

$2.1 \times 10^{-3} - .81 \times 10^{-3} = 1.29 \times 10^{-3}$

2. [5]Simplify. Express results with positive exponents only:
   a. $\frac{(3xy^2)^{-3}}{-x^5y^4}$

$\frac{3^{-3}x^{-3}y^{-6}}{-x^5y^4} = \frac{-1}{3^3x^3y^6x^5y^4} = \frac{-1}{27x^8y^{10}}$

3. [7]Perform the indicated operations and simplify. Be sure to write your answer in the correct form:
   a. $(4x^3 + 7x - 1) \div (2x + 3)$

$2x^2 - 3x + 8 - \frac{25}{2x+3}$

$2x+3\overline{)4x^3 + 0x^2 + 7x - 1}$

$-(4x^3 + 6x^2)$

$-6x^2 + 7x - 1$

$-(-6x^2 - 9x)$

$16x - 1$

$-(16x + 24)$

-25

check

$4(-\frac{3}{2})^3 + 7(-\frac{3}{2}) - 1$

$-4(\frac{27}{8}) - \frac{21}{2} - 1$

$-\frac{27}{2} - \frac{21}{2} - \frac{2}{2}$

$= -\frac{50}{2} = -25$ ✓

$2 \cdot 10 = 20$

1, 20	-1, -20
2, 10	-2, -10
4, 5	-4, -5

4. [15]Factor each polynomial completely:

a. $2x^4 - 9x^3 + 10x^2$

$= x^2(2x^2 - 9x + 10)$

$= x^2(2x-5)(x-2)$

$2x^2 - 4x - 5x + 10$

$2x(x-2) - 5(x-2)$

$(2x-5)(x-2)$

b. $x^4 - 81$

$= (x^2-9)(x^2+9)$

$= (x-3)(x+3)(x^2+9)$

c. $-x^3 + 8$

$= (-x+2)(x^2 - (-2x) + 4)$

$= (-x+2)(x^2 + 2x + 4)$

$-x^3 + 8$

OR $= 8 - x^3$

$= (2-x)(4 + 2x + x^2)$

equivalent

5. [5]Perform the indicated operation and simplify:

a. $(4 - 3i)^2$

$= (4-3i)(4-3i)$

$= 16 - 12i - 12i + 9i^2$

$= 16 - 24i + 9(-1)$

$= 7 - 24i$

6. [5]Simplify and solve for x. Give an exact answer:

a. $x = \log_3 \frac{1}{9}$

$\log_a y = x \iff a^x = y$

$3^x = \frac{1}{9}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

So $x = -2$

Section 2: Graphing and calculations. Show all work to receive credit. Be sure to simplify.

7. [9]Considering the following function: $f(x) = \sqrt{1-3x}$

   a. Find the domain of f(x):

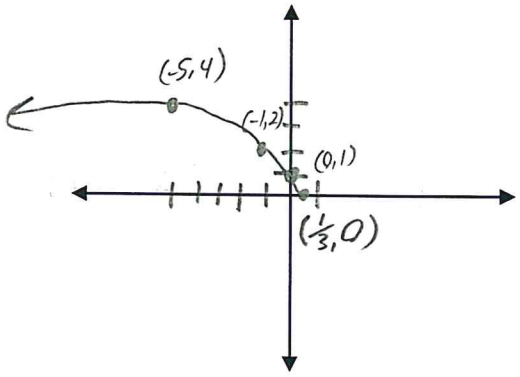
$1-3x \geq 0$ OR $-3x \geq -1$

$1 \geq 3x$ $x \leq -\frac{1}{-3}$

$\frac{1}{3} \geq x$ ← equivalent → $x \leq \frac{1}{3}$

   b. Graph f(x) and **label** at least three points including the x-intercept:

x	$f(x)$
0	$\sqrt{1-3(0)} = \sqrt{1} = 1$
-1	$\sqrt{1-3(-1)} = \sqrt{4} = 2$
-5	$\sqrt{1-3(-5)} = \sqrt{16} = 4$
$\frac{1}{3}$	$\sqrt{1-3(\frac{1}{3})} = \sqrt{0} = 0$



8. [10]Give the equation of the line in slope-intercept form which is perpendicular to the line 2x-4y=7 and passes through the point (-1,3).

put into $y=mx+b$ form

$2x - 4y = 7$

$-4y = -2x + 7$

$y = \frac{1}{2}x - \frac{7}{4}$

slope

Find ⊥ slope

$\frac{1}{2} \longrightarrow -2$

use -2 for slope m and (-1,3) for pt (x,y) in $y - y_1 = m(x - x_1)$

$y - 3 = -2(x+1)$

$y = -2x - 2 + 3$

$y = -2x + 1$

OR

$y = -2x + b$

plug in (-1,3) to find b

$3 = -2(-1) + b$

$b = 3 - 2 = 1$

so $y = -2x + 1$

9. [8]Solve the system of equations:

a. ① $4x-9y=5$
② $-12x+27y=-15$

mult ① by 3 $\quad 3(4x-9y)=3(5)$

$12x-27y=15$

add to ② $\quad -12x+27y=-15$

$0x+0y=0$

$0=0$ always true

infinite solns

10. [6]If y varies inversely as x^2 and y=6 when x=3, find y when x=5. Give an exact answer:

$y=\frac{K}{x^2}$

$6=\frac{K}{3^2}=\frac{K}{9} \longrightarrow 6\cdot 9=K=54$

$y=\frac{54}{5^2}=\frac{54}{25}$

11. [6]Perform the indicated operations, factor, and reduce the following to simplest form. You may leave your answer in factored form:

a. $\left(\frac{x^2-x}{3x+9}\right)\div\left(\frac{x^2-2x+1}{x^2-9}\right)$

$=\frac{x(x-1)}{3(x+3)}\cdot\frac{(x-3)(x+3)}{(x-1)(x-1)}$

$=\frac{x(x-3)}{3(x-1)}$

12. [20]Solve the following equations for x. Give exact answers:

a. $\frac{1}{x+3} - \frac{3}{2x^2+6x} = \frac{5}{2x}$

LCD $= 2x(x+3)$

Mult both sides of = by LCD and cancel $\Rightarrow \frac{(2x)(x+3)(1)}{(x+3)} - \frac{(2x)(x+3)(3)}{(2x)(x+3)} = \frac{(5)}{(2x)}(2x)(x+3)$

$\Rightarrow (2x)(1) - 3 = (5)(x+3)$

$\Rightarrow 2x - 3 = 5x + 15$

$\Rightarrow -18 = 3x$

$\Rightarrow x = -6$

check $\rightarrow \frac{1}{-3} - \frac{3}{+2-36} = \frac{5}{-12}$

$\rightarrow \frac{1}{-3} - \frac{3}{36} = \frac{5}{-12}$

$\rightarrow \frac{-12}{36} - \frac{3}{36} = \frac{-15}{36}$

$\rightarrow \frac{-15}{36} = \frac{-15}{36}$ ✓

b. $y - x = 2ax$

$y = 2ax + x$

$y = x(2a+1)$

$\frac{y}{2a+1} = x$

c. $2x^2 + 3 = 8x$

$2x^2 - 8x + 3 = 0$

$a=2 \quad b=-8 \quad c=3$

$x = \frac{8 \pm \sqrt{64 - 4(2)(3)}}{2(2)} = \frac{8 \pm \sqrt{64-24}}{4} = \frac{8 \pm \sqrt{40}}{4}$

$= \frac{8 \pm 2\sqrt{10}}{4} = \frac{2(4 \pm \sqrt{10})}{4} = \frac{4 \pm \sqrt{10}}{2}$

OR

$2 \pm \frac{\sqrt{10}}{2}$

Section 3: Word problems. Show all work to receive credit. Be sure to use appropriate rounding and include units!

13. [12]The speed of a small plane is 168.0 mph in still air. With a head wind, it makes a routine flight in 2.1 hours. The return flight along the same flight plan with a tail wind takes 1.7 hours. What is the average wind speed? Round to one decimal place.

$d = r \cdot t \qquad d_1 = d_2 \qquad r_1 = 168.0 - w \qquad r_2 = 168.0 + w$

$t_1 = 2.1 \qquad t_2 = 1.7$

$d_1 = (168.0 - w)(2.1) = (168.0 + w)(1.7) = d_2$

$352.8 - 2.1w = 285.6 + 1.7w$

$67.2 = 3.8w$

$w = 17.7 \text{ mph}$

14. [12]How many grams of a 16% tin solder must be mixed with 140 grams of a 38% tin solder to produce a 28% in solder? Round to one decimal place.

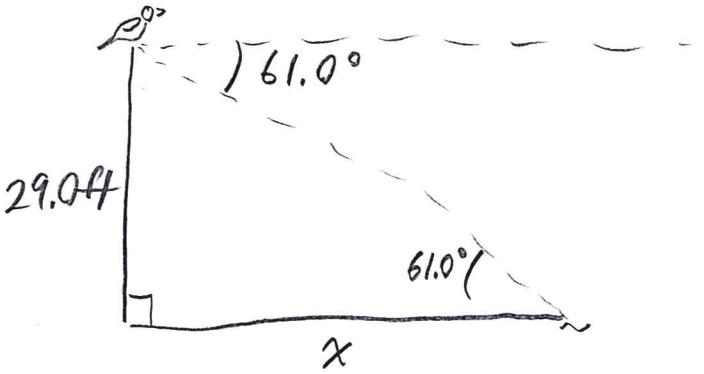
$.16x + .38(140) = .28(x + 140)$

$.16x + 53.2 = 39.2 + .28x$

$14.0 = .12x$

$116.7 g = x$

15. [10]A bird is perched at the top of a tree. On the ground he sees a worm. If the tree is 29.0 feet tall and the angle of depression from the bird to the worm is 61.0°, how far is the worm from the base of the tree? Round appropriately.

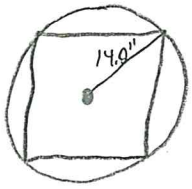


$\tan(61.0) = \frac{29.0}{x}$

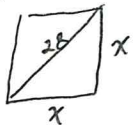
$x = 16.1 \text{ ft}$

Section 4: Geometry and Trig. Show all work to receive credit. Round appropriately or as indicated in the problem.

16. [10]Find the area of the following:
    a. The area between a circle and an inscribed square where the circle has radius 14.0in. Round to three significant digits.



Area □ ?



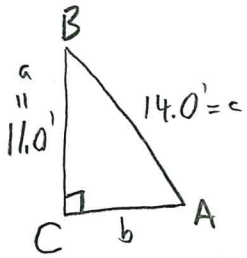
$x^2 + x^2 = 28^2 \quad 2x^2 = 28^2$

$\text{Area } \square = x^2 = \frac{28^2}{2} = 392 \text{ in}^2$

$\text{Area } O = \pi r^2 = \pi(14)^2 = 615.75 \text{ in}^2$

$\text{Area } O - \text{Area } \square = 223.75 \text{ in}^2 \rightarrow 224 \text{ in}^2$

17. [12]Given the three parts of a triangle, find the remaining three parts. You may round off the length of sides to three significant digits and round off angles to the nearest 1/10 of a degree.
    a. C=90.0°, a=11.0°, c=14.0 ft



b = 8.66ft typo

A = 51.8°

B = 38.2°

$a^2 + b^2 = c^2$

$11^2 + b^2 = 14^2$

$b = 8.66 \text{ ft}$

$\sin A = \frac{11.0}{14.0}$

$A = 51.8°$

$B = 180° - 90° - 51.8° = 38.2°$

18. [12]Find the area of a regular octagon with perimeter of 88.0 feet.

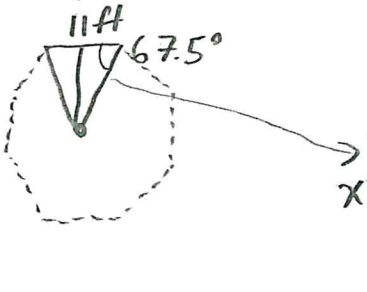
8 sides

$\frac{88.0}{8} = 11.0 \text{ ft/side}$

Sum Interior angles

$(n-2)(180) = 6(180) = 1080$

$\frac{1080}{8} = 135°/\text{interior angle}$



Need to find x

$\tan(67.5°) = \frac{x}{5.5}$

$x = 13.278 \text{ ft}$

$\text{Area } \triangle = \frac{1}{2} b \cdot h$

$= \frac{1}{2}(5.5)(13.278)$

$= 36.515 \text{ ft}^2$

$\text{Area } \text{⯃} = 16 \cdot \text{Area } \triangle$

$= 16(36.515)$

$= 584 \text{ ft}^2$

$A_s = \frac{\theta}{2} r^2$

19. [8]Find the area of the sector of a circle that has a diameter of 10.0 cm and a central angle of 115°. Round to three significant digits.

radius = 5.0cm = r

$\frac{115°}{180°} \left| \frac{\pi \text{ radians}}{} \right| = 2.007 \text{ radians} = \theta$

$A_s = \frac{2.007}{2}(25.9) = 25.1 \text{cm}^2$

20. [15]Given a=58.4m, b=66.2m, c = 27.1m, find the three remaining parts of an oblique triangle. You may round off the length of sides to the correct number of significant figures and round off angles to the nearest 1/10 of a degree:

Hint: Law of Sines: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$, Law of Cosines: $\begin{cases} a^2 = b^2 + c^2 - 2bc(cosA) \\ b^2 = a^2 + c^2 - 2ac(cosB) \\ c^2 = a^2 + b^2 - 2ab(cosC) \end{cases}$

Hint: Find Angle A first.

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$A = 61.6°$

Note: If you use Law of Sines to find B, you will get the wrong Δ. You can, however, use it to find C and then calculate B.

$b^2 = a^2 + c^2 - 2ac(\cos B)$

$B = 94.3°$

$C = 180° - 94.3° - 61.6° = 24.1°$

21. [8]Sketch a graph of $f(x) = 2cosx - 1$. Be sure to show at least one full period. Label the y-intercept and three other points.