Math 125 Final Exam, Spring 2019

- The following exam has 4 parts, 19 problems, and 8 pages. Please stop and make sure that your exam has all its pages.
- Please raise your hand if you have any questions or need a restroom break.
- When you have completed your exam, raise your hand and the instructor will collect your exam. DO NOT begin packing up until you have turned your exam in.
- ANY use of cell phones or electronics other than an appropriate calculator will result in you receiving a zero on your final exam.
- ANY cheating (cheat sheets, communicating with classmates, etc.) will result in you receiving a zero on your final exam.

Section 1: Quick problems. Show work to receive partial credit. Make sure you simplify fully and round appropriately.

1. [5]Complete the indicated operation and give your answer in scientific notation. Round appropriately as your final step:

   a. $1.3 \times 10^{-3} + 3.8 \times 10^{-4}$

   $$\begin{array}{r} .0013 \\ + \ .00038 \\ \hline .00168 \end{array}$$

   $1.68 \times 10^{-3} \longrightarrow 1.7 \times 10^{-3}$

2. [5]Simplify. Express results with positive exponents only:

   a. $\left(\frac{3b^5}{y^3}\right)^{-1}$

   $\dfrac{3^{-1}b^{-5}}{y^{-3}} \qquad \dfrac{y^3}{3b^5}$

3. [7]Perform the indicated operations and simplify. Be sure to write your answer in the correct form:

   a. $(8x^2 - 7 + 10x) \div (2x + 1)$

   $$\begin{array}{r} 4x + 3 \quad -\frac{10}{2x+1} \\ 2x+1 \overline{)\ 8x^2 + 10x - 7} \\ -(8x^2 + 4x) \\ \hline 6x - 7 \\ -(6x + 3) \\ \hline -10 \end{array}$$

a.

$- 48$

-1	48
1	-48
-2	24
2	-24
-3	16
3	-16

4. [15]Factor each polynomial completely:

a. $24x^2 + 13x - 2$

$$24x^2 - 3x + 16x - 2$$
$$3x(8x - 1) + 2(8x - 1)$$
$$(3x + 2)(8x - 1)$$

b. $x^4 - 81$

$$(x^2 - 9)(x^2 + 9)$$
$$(x - 3)(x + 3)(x^2 + 9)$$

c. $x^3 + 27$

$$(x + 3)(x^2 - 3x + 9)$$

Section 2: Graphing and calculations. Show all work to receive credit. Be sure to simplify.

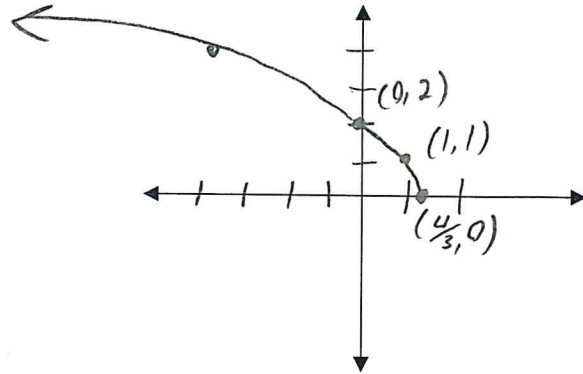5. [9]Considering the following function: $g(x) = \sqrt{4 - 3x}$
   a. Find the domain of g(x):

   $4 - 3x \geq 0 \qquad 4 \geq 3x \qquad \frac{4}{3} \geq x$

   $\left(-\infty, \frac{4}{3}\right]$

   b. Graph g(x) and **label** at least three points including the x-intercept:

x	y
$\frac{4}{3}$	0
1	1
0	2
-4	4

   (0, 2)
   (1, 1)
   $\left(\frac{4}{3}, 0\right)$

6. [10] Give the equation in slope-intercept form of the line that is perpendicular to the line $y = \frac{1}{2}x - 5$ and passes through the point (3,0):

   perpendicular to $\frac{1}{2} \longrightarrow -2 = m$

   $y - y_1 = m(x - x_1)$

   $y - 0 = -2(x - 3)$

   $y = -2x + 6$

   OR

   $(3, 0) = (x_1, y_1)$

   $y = -2x + b$

   $0 = -6 + b$

   $6 = b$

   $y = -2x + 6$

7. [8]Solve the system of equations:

    a. $4x=8-3y$
        $8x+6y=6$

    $-2(4x+3y=8)$
    $\quad 8x+6y=6$

    $-8x-6y=-16$
    $\ \ 8x+6y=6$
    $\qquad 0=-10$ parallel lines
    $\quad$ No soln

8. [6] If y varies directly with x^2 and y=30 when x=5, find y when x=3. Give an exact answer:

    $y=Kx^2$

    $30=K(5)^2$

    $\dfrac{30}{25}=K$

    $\dfrac{6}{5}=K$

    $y=\dfrac{6}{5}(3)^2$

    $y=\dfrac{6}{5}(9)$

    $y=\dfrac{54}{5}$

9. [6]Perform the indicated operations, factor, and reduce the following to simplest form. You may leave your answer in factored form:

    a. $\left(\dfrac{x^2-16}{x}\right) \div \left(\dfrac{x+4}{2x^2-14x}\right)$

    $\dfrac{x^2-16}{x} \cdot \dfrac{2x^2-14x}{x+4}$

    $\dfrac{(x-4)(x+4)}{x} \cdot \dfrac{2x(x-7)}{x+4}$

    $(x-4) \cdot 2(x-7)$

    $2(x-4)(x-7)$

10. [24]Solve the following equations for x. Give exact answers:

a. $\frac{7}{2x} + \frac{1}{9} = \frac{3}{6x}$ LCD $18x = 2 \cdot 3^2 \cdot x$

$2 \cdot x \quad 3^2 \quad 2 \cdot 3 \cdot x$

$\overset{9}{18x}\left(\frac{7}{2x}\right) + \overset{2}{18x}\left(\frac{1}{9}\right) = \overset{3}{18x}\left(\frac{3}{6x}\right)$

$9(7) + 2x(1) = 3(3)$

$63 + 2x = 9$

$2x = -54$

$x = -27$

b. $K = \frac{ax}{x+b}$

$K(x+b) = ax$

$Kx + Kb = ax$

$Kx - ax = -Kb$

$x(K-a) = -Kb$

$x = \frac{-Kb}{K-a}$

or

$Kb = ax - Kx$

$Kb = x(a-k)$

$\frac{Kb}{a-k} = x$

$\xleftarrow{equiv}$

c. $x^2 - 2 = -7x$

$x^2 + 7x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{49 - 4(1)(-2)}}{2(1)}$
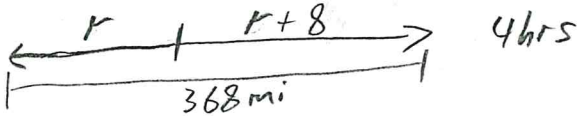
$x = \frac{-7 \pm \sqrt{57}}{2}$

Section 3: Word problems. Show all work to receive credit. Be sure to use appropriate rounding and include units!

11. [12] Two trucks leave a warehouse at the same time, travelling in opposite directions. The speed of the faster truck is 8.0 mph faster than the speed of the slower truck. 4.0 hours later, the trucks are 368 miles apart. What is the speed of the slower truck? Round your answer to a whole number.

$d = r \cdot t$

$d = d_1 + d_2 = 368 \, mi$

$r_1 = r \quad r_2 = r + 8$

$t_1 = t_2 = 4 \, hrs$

$$d = d_1 + d_2 = (r_1)(t_1) + (r_2)(t_2)$$
$$d = 368mi = (r)(4) + (r+8)(4)$$
$$368 = 4r + 4r + 32$$
$$368 = 8r + 32$$
$$\boxed{42mph = r}$$

12. [12] How many liters of a 30% acid solution must be mixed with a 15% acid solution to produce 10 liters of a 17% acid solution? Round to one decimal place.

$x + y = 10$
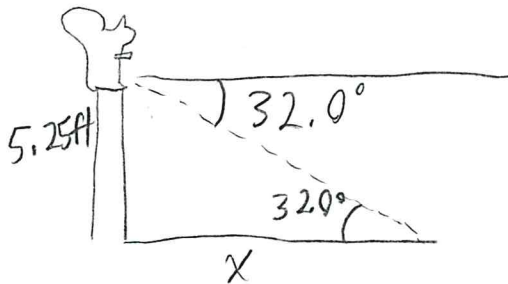
$y = 10 - x \longrightarrow$

$$.3x + .15y = .17(10)$$
$$.3x + .15(10-x) = .17(10)$$
$$.3x + 1.5 - .15x = 1.7$$
$$.3x - .15x = 0.2$$
$$.15x = 0.2$$
$$x = 1.3 \, L$$

13. [12] A squirrel stands on the top of a fence post. On the ground he sees an acorn. If the fence post is 5.25 feet tall and the angle of depression from the squirrel to the acorn is 32.0°, how far is the acorn from the base of the post? Round appropriately.
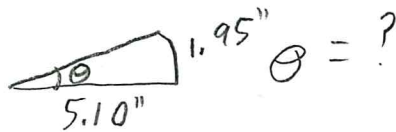
$$\tan 32.0° = \frac{5.25}{x}$$

$$x = \frac{5.25}{\tan(32.0°)}$$

$$x = 8.40 \, ft$$

17. [10]Find the measure of the **central** angle in degrees for a sector of a circle that has a radius of 5.10 inches and an **arc length** of 1.95 inches. Round to three significant digits.
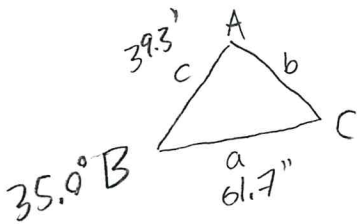
$$S = \theta r$$

$$\theta = \frac{S}{r} = \frac{1.95"}{5.10"} = 0.382\,rad$$

$$\left|\frac{0.382\,rad}{1}\right|\left|\frac{180°}{\pi\,rad}\right| = 21.9°$$

$\theta = ?$

18. [15] Given a=61.7in, c= 39.3in, B=35.0°, find the three remaining parts of an oblique triangle. You may round off the length of sides to the correct number of significant figures and round off angles to the nearest 1/10 of a degree:

Hint: Law of Sines: $\dfrac{a}{sinA} = \dfrac{b}{sinB} = \dfrac{c}{sinC}$, Law of Cosines: $\begin{cases} a^2 = b^2 + c^2 - 2bc(cosA) \\ b^2 = a^2 + c^2 - 2ac(cosB) \\ c^2 = a^2 + b^2 - 2ab(cosC) \end{cases}$

$b = 37.1"$
$A = 107.6"$
$C = 37.4"$

$$b^2 = a^2 + c^2 - 2ac\,(cosB)$$
$$b^2 = 61.7^2 + 39.3^2 - 2(61.7)(39.3)(cos(35.0))$$
$$b^2 = 1378.803863$$
$$b = 37.1"$$
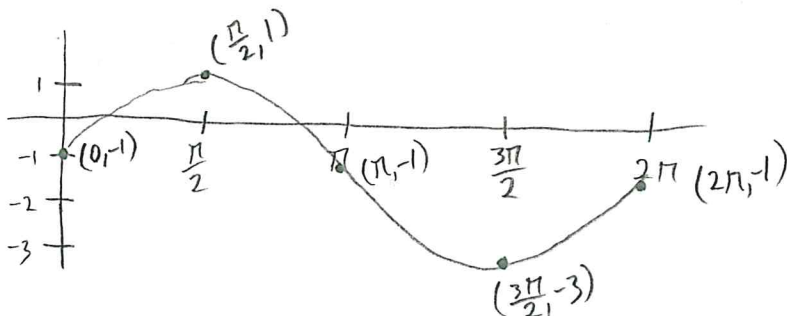
$$\frac{sinC}{c} = \frac{sinB}{b}$$

$$sinC = (39.3)\frac{sin(35.0)}{37.1} \qquad C = 37.4°$$

$$A = 180 - A - C = 107.6$$

19. [10]Sketch a graph of $f(x) = 2sinx - 1$. Be sure to show at least one full period. Label the y-intercept and three other points.

Section 4: Geometry and Trig. Show all work to receive credit. Round appropriately or as indicated in the problem.
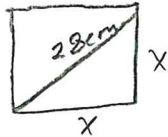
14. [10]Find the area of the following:
   a. The area between a circle and an inscribed square where the circle has radius 14cm. Round to three significant digits.
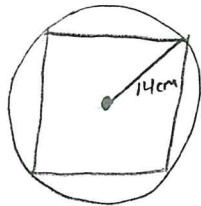
$$A_o = \pi r^2 = 615.7521601 cm^2$$
$$A_\square = b \cdot h = b^2 = 392 cm^2$$
$$A_o - A_\square = 224 cm^2$$

$$a^2 + b^2 = c^2$$
$$x^2 + x^2 = 28^2$$
$$2x^2 = 28^2 \rightarrow x^2 = 392$$

15. [12]Given the three parts of a triangle, find the remaining three parts. You may round off the length of sides to three significant digits and round off angles to the nearest 1/10 of a degree.
   a. C=90°, A=77°, c=145 cm $a = 141 cm$ $B = 13.0°$ $b = 32.6 cm$

$$\sin A = \frac{a}{c} \rightarrow \sin 77° = \frac{a}{145} \rightarrow a = 141 km$$
$$\cos A = \frac{b}{c} \rightarrow \cos 77° = \frac{b}{145} \rightarrow b = 32.6 cm$$
$$B = 180 - A - C \rightarrow B = 180 - 77 - 90$$
$$\rightarrow B = 13.0°$$

16. [12] Find the area of a regular **hexagon** with perimeter of 746 feet.
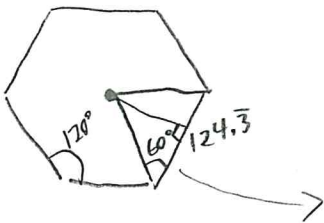   6 sides

$$\frac{746}{6} = 124.3$$

Sum interior $\angle$
$(n-2)(180°)$
$4(180°) = 720°$
$\frac{720}{6} = 120°$

$$\tan 60° = \frac{h}{62.16} \rightarrow h = 107.6758252$$

$$A_\triangle = \frac{b \cdot h}{2} = 3346.923567$$
$$A_O = 12 \cdot A_\triangle = 40163.08 \rightarrow 40200 ft$$

62.16