TURN OFF YOUR CELL PHONE AND PUT IN BAG. IF YOUR CELL PHONE IS SEEN, YOU WILL EARN A "0". SHOW ALL WORK CLEARLY FOR CREDIT. SCIENTIFIC CALCULATOR ONLY!!

[6] 1. Consider: $f(x) = \sqrt{2x-5}$

a) Determine of the domain of $f(x)$
Use interval notation.

$2x-5 \geq 0$

$x \geq 5/2$

$[5/2, \infty)$

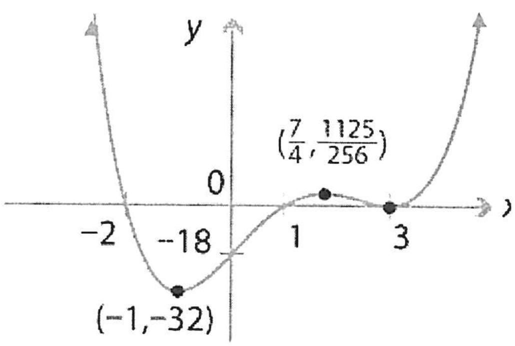
b) Solve : $f(x) = 4$ algebraically.

$\sqrt{2x-5} = 4$

$2x-5 = 16$

$2x = 21$ $\quad x = 21/2$

[12] 2. Use the graph to answer the questions:




a) Give the interval(s) on which y=f(x) is increasing.

$(-1, 7/4) \cup (3, \infty)$

b) Give the coordinates of the relative extrema or write none.

Relative min: $(-1,-32)$ & $(3,0)$

Relative max: $(7/4, \frac{1125}{256})$

c) Give the domain (in interval notation)

$(-\infty, \infty)$

d) Give the range in interval notation.

$[-32, \infty)$

e) Give the zeros. For each zero, indicate if the multiplicity would be even or odd.

-2 odd

1 odd

3 even

f) Is the degree of this polynomial even or odd? Explain.

even ↑ ↑ end behavior.

[7] 3. If $g(x) = -x^2 + 6x$, find: $\dfrac{g(x+h)-g(x)}{h}$

$$\frac{-(x+h)^2 + 6(x+h) - (-x^2+6x)}{h}$$

$$= \frac{-(x^2+2xh+h^2) + 6x + 6h + x^2 - 6x}{h}$$

$$= \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{6x} + 6h + \cancel{x^2} - \cancel{6x}}{h}$$

$$= \frac{\cancel{h}(-2x-h+6)}{\cancel{h}}$$

$$= \boxed{-2x-h+6}$$

[9] 4. Fill in the chart with EXACT values. Also include the radian measure.

| | x = $30^0$ or $\pi/6$ radians | x = $45^0$ or $\pi/4$ radians | x= $60^0$ or $\pi/3$ radians |
|---|---|---|---|
| sin(x) | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ |
| cos(x) | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ |
| tan(x) | $1/\sqrt{3}$ | 1 | $\sqrt{3}$. |

[21] 5. Solve each equation. Use exact values (**no calculators/decimals**)

a) $2cos^2\theta - cos\theta - 1 = 0$ on the interval $[0, 360^\circ)$

$(2\cos\theta+1)(\cos\theta-1)=0$

$\cos\theta = -1/2$ $\cos\theta = 1$

$\theta' = 60^\circ$

$\boxed{120^\circ, 240^\circ, 0^\circ}$

$(1,0)$

b) $\sin(2\theta) = \frac{\sqrt{3}}{2}$ on the interval $0 \le \theta < 2\pi$

$0 \le 2\theta < 4\pi$

$2\theta = \pi/3$

$2\theta = 2\pi/3$

$2\theta = 7\pi/3$

$2\theta = 8\pi/3$

$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}}$

c) Find the general solution (all solutions) to $\sin(x)\tan(x) = \sin(x)$. Use radians.

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$ $\tan x = 1$

$n\pi$

$5\pi/4$ $\pi/4$

$n\pi$

$\frac{\pi}{4} + n\pi$ n an integer

$\frac{5\pi}{4} + n\pi$ or $\boxed{n\pi \quad \frac{\pi}{4} + n\pi}$

[ 10] 6. Given $g(x) = 2x^3 + x^2 - 8x - 4$

a) Give the y-intercept $(0,-4)$

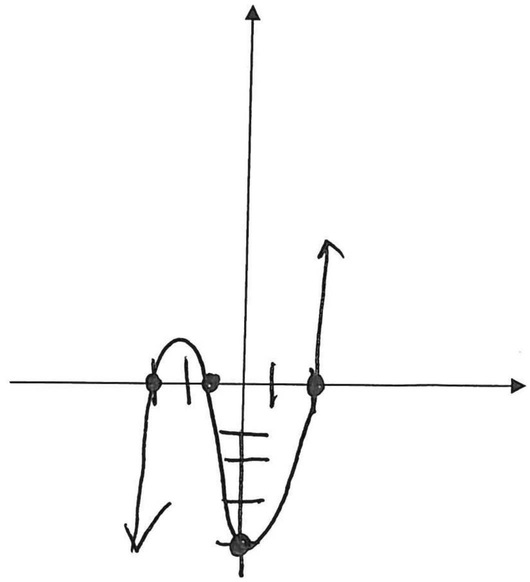
b) Give the factored form

$x^2(2x+1) - 4(2x+1)$

$(2x+1)(x^2-4) = (2x+1)(x+2)(x-2)$

c)

| Zeros | Multiplicity | Tangent or cross through? |
|---|---|---|
| $-2$ | 1 | cross |
| $+2$ | 1 | cross |
| $-\frac{1}{2}$ | 1 | cross |



d) Draw the end behavior:

e) Graph using a-d. Label all intercepts!

[12] 7. Consider $g(x) = \frac{2x+6}{x^2+x-6}$ $\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}$

a) State the domain of f(x) (any notation is fine)

$x \neq 2, -3$

b) Find the y-intercept or state none

$(0,-1)$

c) Find the vertical asymptote (or state none)

$x = 2$

d) Find the zero(s) or state none

none

e) Find the horizontal asymptote (or state none)

$y = 0$

f) Find the x and y-coordinates any holes (or state none).

$y = \frac{2}{-3-2}$

$(-3, -\frac{2}{5})$

[10] 8. Graph. Fill in information, showing work algebraically. Label vertex, zeros and intercept on graph

$y = x^2 + 6x - 7$

$x = -\frac{6}{2(1)} = -3$

Vertex: $(-3,-16)$

$y = (-3)^2 + 6(-3) - 7$

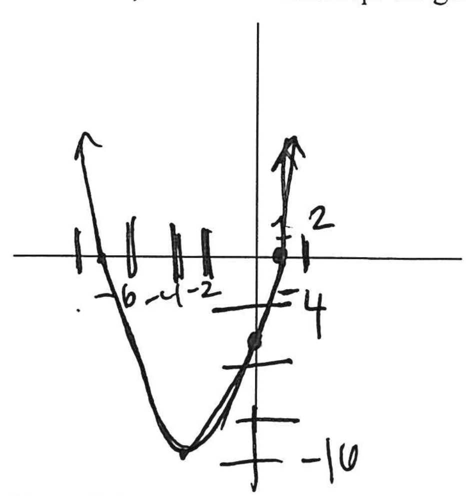
Zeros: $-7, 1$

$(x+7)(x-1) = 0$

$x = -7, +1$

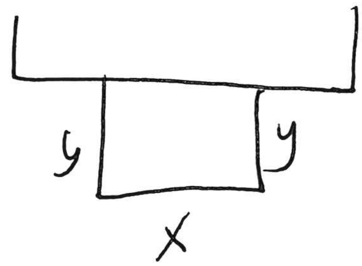
y-intercept:

$(0,-7)$



[8] 9. A class wants to enclose a rectangular garden using 75 feet of fence. The side of the school is used as one side of the rectangle (thus fencing is only needed on 3 sides). Draw a picture and label the sides with variables.

a) Find a function for the area of the garden in one variable.




$A = xy$

$A = (75-2y)(y)$ $\quad x+2y=75$

$x = 75-2y$

b) What dimensions (length and width) yield maximum area? **Show your work algebraically** or no credit will be given. Put units on your answer.

$A = 75y - 2y^2$

$y = \frac{-75}{2(-2)} = \underline{18.75\text{ ft}}$ $\quad x = 75 - 2(18.75)$

$= \underline{37.5\text{ ft}}$

[7] 10. Find the inverse. Show all work algebraically: $g(x) = \frac{2x-1}{3x+5}$

$x = \frac{2y-1}{3y+5}$

$3xy + 5x = 2y - 1$

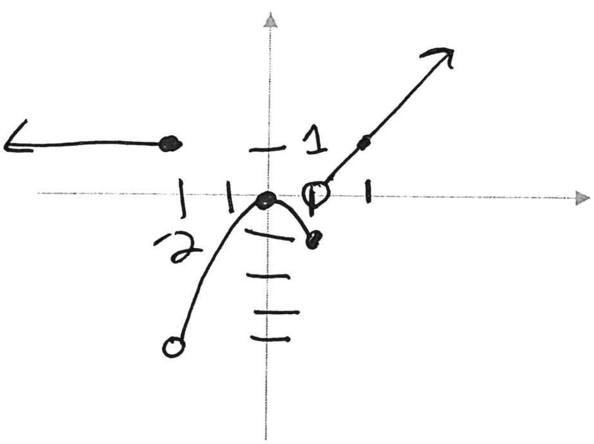
$3xy - 2y = -1 - 5x$

$y(3x-2) = -1-5x$

$y = \frac{-1-5x}{3x-2}$

$\boxed{f^{-1}(x) = \frac{-1-5x}{3x-2}}$

[7] 11. Graph: $f(x) = \begin{cases} 1, & x \le -2 \\ -x^2, & -2 < x \le 1 \\ x-1, & x > 1 \end{cases}$ Mark endpoints open or closed.



[28] 12. Solve. Give exact answers and show work algebraically. **Do not use decimals.**

a) $x^4 - 8x^2 - 9 \geq 0$. Write solution in interval notation.

$(x^2-9)(x^2+1) \geq 0$

$x = \pm 3$ $\quad \cancel{x^2 = -1}$

yes NO yes

-3 3

$(-4)^4 - 8(-4)^2 - 9 > 0$ yes

$0^4 - 8(0)^2 - 9 \geq 0$ NO.

$4^4 - 8(4)^2 - 9 \geq 0$ yes

$\boxed{(-\infty, -3] \cup [3, \infty)}$

b) $e^x - 14e^{-x} + 5 = 0$

$\left(e^x - \frac{14}{e^x} + 5 = 0\right) e^x$

$e^{2x} - 14 + 5e^x = 0$

$e^{2x} + 5e^x - 14 = 0$

$(e^x + 7)(e^x - 2) = 0$

$e^x = -7$ NO.

$e^x = 2$

$\ln e^x = \ln 2$

$\boxed{x = \ln 2}$

c) $12\ln(5x + 1) = 4$

$\ln(5x+1) = \frac{1}{3}$

$e^{1/3} = 5x + 1$

$e^{1/3} - 1 = 5x$

$\boxed{\frac{e^{1/3} - 1}{5} = x}$

d) $2\left|\frac{1}{3}x - 4\right| + 3 = 9$

$2\left|\frac{1}{3}x - 4\right| = 6$

$\left|\frac{1}{3}x - 4\right| = 3$

$\frac{1}{3}x - 4 = 3$

$\frac{1}{3}x = 7$

$x = 21$

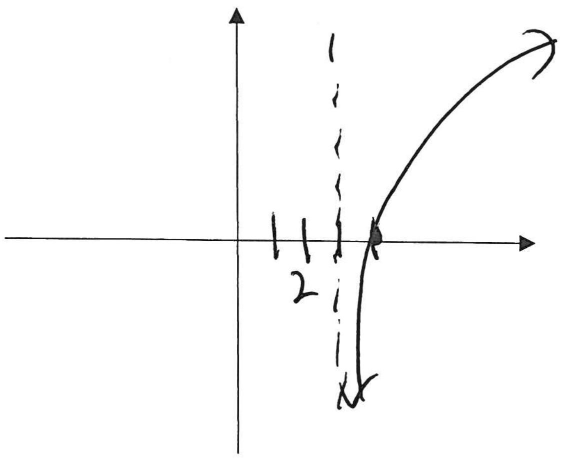
$\frac{1}{3}x - 4 = -3$

$\frac{1}{3}x = 1$

$\boxed{x = 3}$

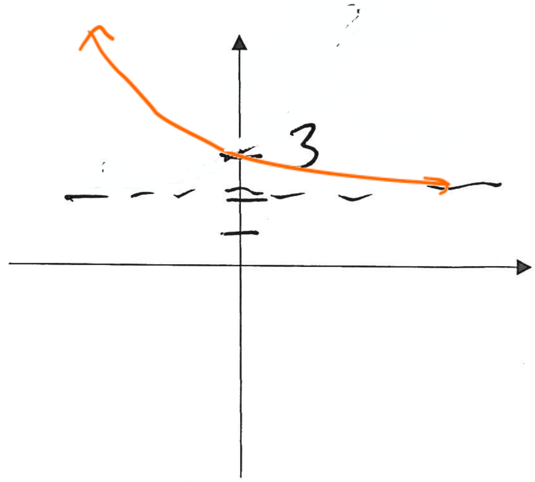
[12] 13. Graph each of the following. Label the indicated intercept and asymptote. Dash in Asymptote.

a) $f(x) = log_3(x-3)$




x-int: $(4,0)$ VA: $x=3$

b) $f(x) = e^{-x} + 2$




y-int: $(0,3)$ HA: $y=2$

[7] 14. Find the linear function $f$ such that $f(-1) = 5$ *and* $f(2) = 7$ $\Rightarrow (-1,5)\ (2,7)$

Final answer: f(x) = $2/3x + 17/3$

$$m = \frac{7-5}{2-(-1)} = \frac{2}{3}$$

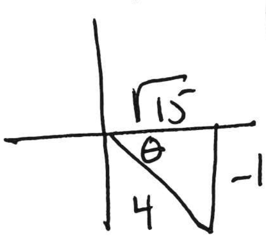
$$y - 5 = \frac{2}{3}(x+1)$$

$$y = \frac{2}{3}x + \frac{2}{3} + 5$$

[9] 15. Suppose $\sin(\theta) = -\frac{1}{4}\ \frac{O}{H}$ where $\theta$ is in Quadrant IV. Find the following. Give exact values (no decimals)

a) $\cos\theta$

$$\frac{a}{h} = \boxed{\frac{\sqrt{15}}{4}}$$




$a^2 + (-1)^2 = 16$

$a^2 = 15$

$a = \sqrt{15}$

b) $2 - 32sin^2\theta$

$$2 - 32\left(-\frac{1}{4}\right)^2$$

$$2 - 32\left(\frac{1}{16}\right)$$

$$2 - 2 = 0$$

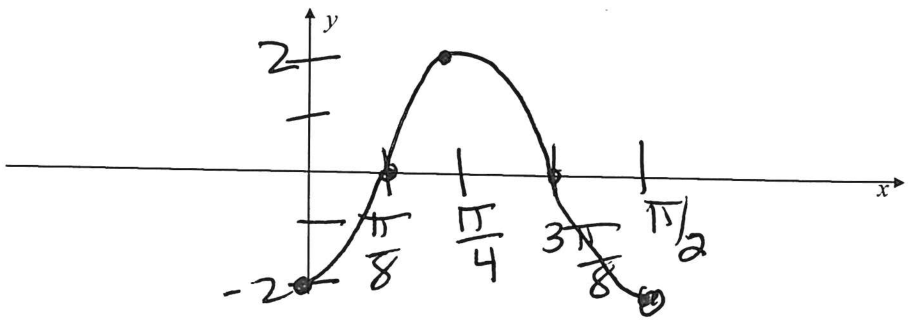
c) $\sin(2\theta)$

$$= 2\sin\theta\cos\theta$$

$$2\left(-\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right)$$

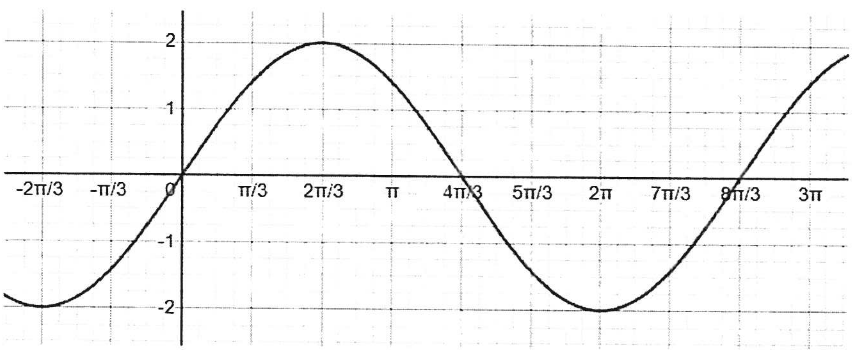
$$= \frac{-\sqrt{15}}{8}$$

[7] 16. Graph at least one period. **Clearly label** each graph pointing out $x$-intercepts and maximum and minimum points. Label at least 4 tick marks on x-axis and at least one tick mark on y-axis.
$y = -2\cos(4x)$




$\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}.$

[7] 17. Given the graph, find the following:




a) domain: $(-\infty, \infty)$

No phase shift or up/down

b) Period: $8\pi/3$

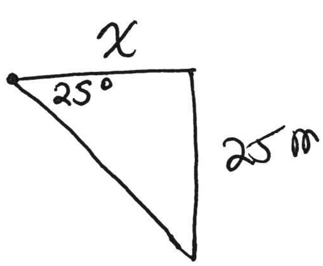
c) Amplitude: 2

d) Equation of this function: $y = 2\sin\left(\frac{3}{4}x\right)$

$\frac{8\pi}{3} = \frac{2\pi}{b}$ $\quad 8\pi b = 6\pi$

$b = \frac{6\pi}{8\pi} = \frac{3}{4}$

[7] 18. From the top of a bridge, Maria looks down at a sailboat at an angle of depression of 25°. The bridge is 25m above the water. Calculate the horizontal distance from the bridge to the sailboat. Round to 2 decimal places and put units on your answer.




$\tan 25° = \frac{25}{x}$

$x \tan 25° = 25$

$x = \frac{25}{\tan 25°} = \boxed{53.61\text{ m}}$

[14] 19. Prove **any 2** of the following 3**. Check the two boxes of the problems you want graded.** Put reasons next to each step. If you don't check two boxes, the first two will be graded, regardless of work.

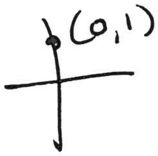
a) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

Grade? ☐

LHS =

$\cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$ diff. ID.

$0 \cdot \cos\theta + 1 \cdot \sin\theta$ Sub.




$= \sin\theta = RHS.$

b) $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

Grade? ☐

$LHS = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x} = \frac{\cancel{\cos x}(1+\sin x)}{\cancel{\cos^2 x}}$

$\cos x$

Pytnag. ID.

$= \frac{1+\sin x}{\cos x}$ (cancel).

c) $\tan\alpha + \cot\alpha = \sec\alpha \csc\alpha$

Grade? ☐

$\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$ Quotient ID.

$\frac{\sin\alpha \cdot \sin\alpha}{\cos\alpha \cdot \sin\alpha} + \frac{\cos\alpha}{\sin\alpha} \cdot \frac{\cos\alpha}{\cos\alpha}$ Common denom.

reciprocal ID

pytnag. ID.

$\frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha} = \frac{1}{\cos\alpha\sin\alpha} = \frac{1}{\cos\alpha} \cdot \frac{1}{\sin\alpha} = \sec\alpha\csc\alpha$

$= RHS.$