MATH 109 – Final Exam
Fall 2021

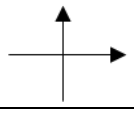
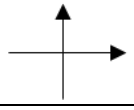
1. [5] State the **domain** and **range** of the following trigonometric functions in interval notation:

Function	Domain	Range
$y = \sin x$		
$y = \cos x$		
$y = \sin^{-1} x$		
$y = \cos^{-1} x$		
$y = \tan^{-1} x$		

2. [5] Find the **<u>least positive coterminal</u>** angle of $\frac{30\pi}{7}$.

3. [5] Find the **<u>supplementary angle</u>** (if any) of $\frac{13\pi}{16}$.

4. [10] Complete the table:

		Quadrant where angle θ terminates	**Reference angle** θ'	**Sign** (+ or –)	**Equivalent form** in terms of reference angle	**Exact Value** (Answer not in decimal)
a)	$\cos 660^\circ$					
b)	$\sec \frac{19\pi}{6}$					

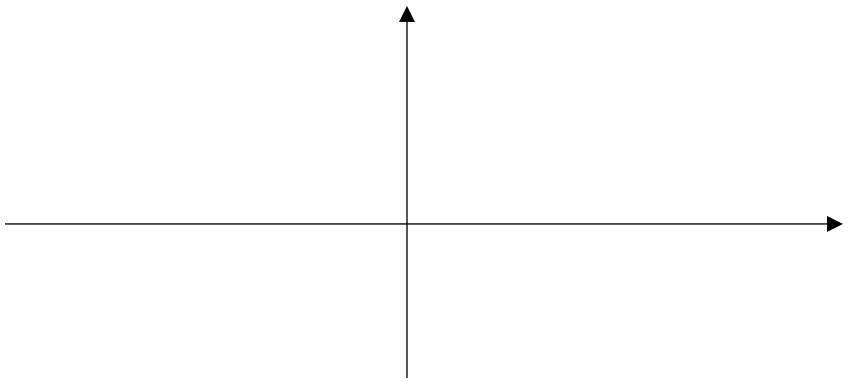
5. [6] If θ is an acute angle and $\cos\theta = \frac{2}{3}$, Find the following:

a) $\sqrt{1-\sin^2\theta}$ b) $\csc\theta$

6. [8] Find the following for $f(x) = -3\sin\left(4x - \frac{\pi}{3}\right) + 2$.

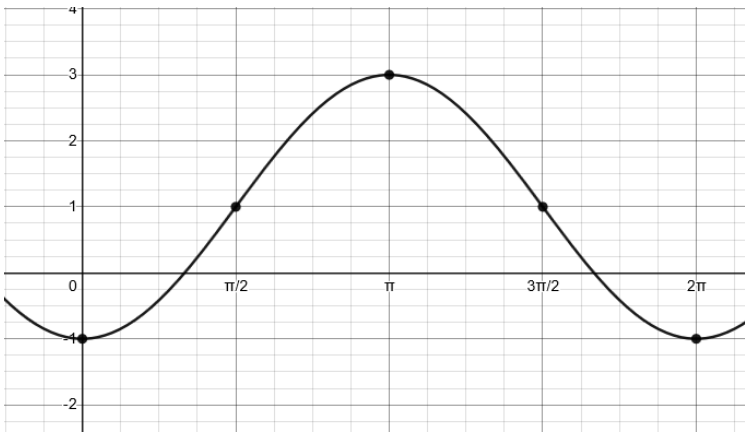
Amplitude:	
Period:	
Phase shift:	
Range:	

7. [6] **Sketch** the graph of $y = 2\sin(4x) + 1$ for **only one period**. **Label** all the tick marks on both x and y axis where necessary.



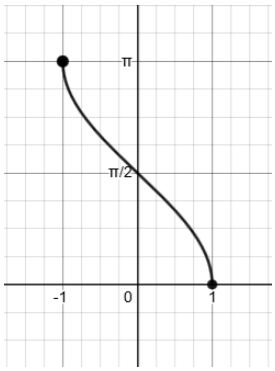
8. [12] Write an equation for the following graphs:

a)



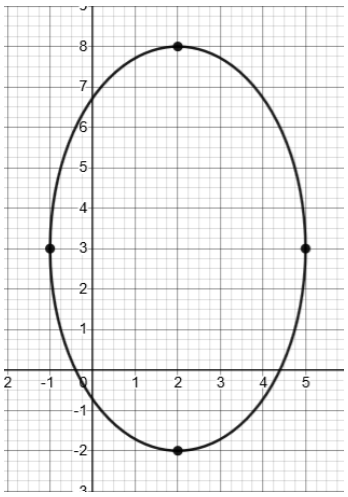
Equation: _____

b)



Equation: _____

c)



Equation: _____

9. [4] Complete the identities (No Partial credit).

$1+\tan^2 x$	$=$
$\sin(2x)$	$=$
$\cos^2 x-\sin^2 x$	$=$
$\cos\left(\frac{\pi}{2}-\theta\right)$	$=$

10. [8] Assuming that $\cos A=-\frac{2}{3}$ where A is in quadrant II and $\sin B=-\frac{1}{3}$ where B is in quadrant IV. Find the **exact value** of $\cos(A-B)$.

11. [6] If $x=3\sec\theta$ then find and simplify $\sqrt{x^2-9}$ given that $0<\theta<\frac{\pi}{2}$.

12. [8] Verify the identity: $\sin\theta\left(\tan\theta+\cot\theta\right)=\sec\theta$

13. [8] Solve: $\sin x=\frac{\sqrt{3}}{2}$ where x is in radians. (**General solutions. No decimal answers**)

14. [8] Solve: $\sin x=-0.25$ where x is in $[0^\circ,360^\circ)$. **Round** your answer(s) **to the nearest degree**.

15. [8] Solve: $2\sin^2 x + \cos x - 1 = 0$ on $[0, 2\pi)$. **(No decimal answers.)**

16. [12] Given $\tan x = -\frac{3}{4}$ and x is in **Quadrant IV**. Find the exact value of the following and **state** the quadrant in which $2x$ lies.

a) [3] $\sin(2x) =$

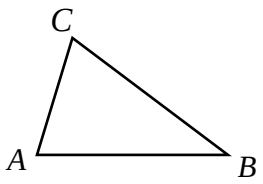
b) [3] $\cos(2x) =$

c) [3] $\tan(2x) =$

d) [3] $2x$ lies in quadrant _____

17. [8] Solve the triangle ABC, if possible. Round your answers to **one decimal place**.

$$a = 17in \ , b = 13in \ , \ A = 75^\circ$$

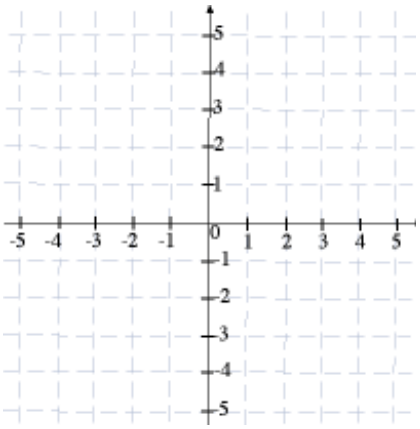


18. [6] Convert the rectangular coordinate $\left(-\sqrt{2}, \sqrt{2}\right)$ to **<u>polar coordinate</u>**. (Exact and **do not use decimals**.)

19. [6] Convert to **<u>rectangular equation</u>**: $r = 3\cos\theta$

20. [12] **Classify** the graph of the equation circle, a parabola, or an ellipse for part a). Find the necessary information as given and write N/A for any that do not pertain to this type of conic and **sketch the graph**:

$$y^2 + 4y + 4x = 0$$



a) Type of Conic: _____

b) Center: _____

c) Vertex or Vertices: _____

d) Focus or Foci: _____

e) Directrix: _____

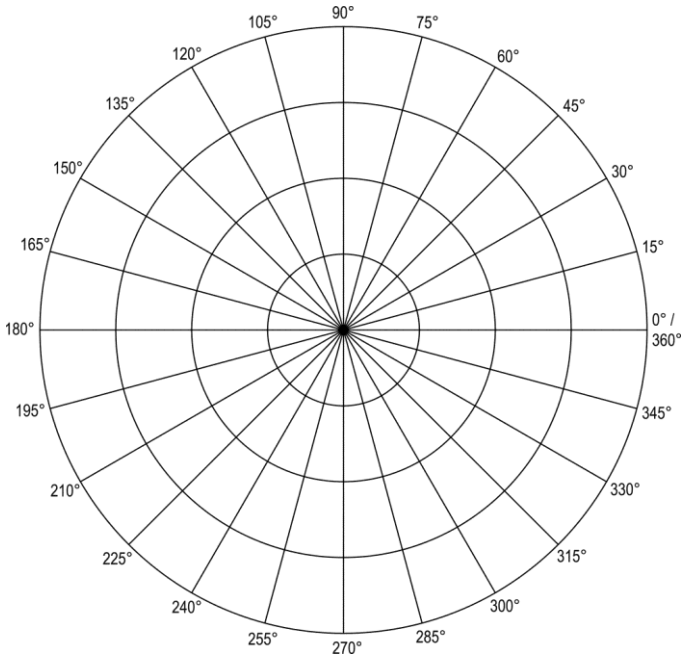
f) Radius: _____

21. [8] Use De Moivre's theorem to raise the complex number to the given power. Write your answer in **standard notation,** $a+bi$.

$$\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^6$$

22. [8] Complete the table (round to one decimal place) and graph $r = 2 - 2\cos\theta$.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r													



23. [8] Perform the following if $\vec{u} = \langle 5, -2 \rangle$ and $\vec{v} = \langle -4, 7 \rangle$

a) $\vec{u} - 2\vec{v}$ b) $\vec{u} \cdot \vec{v}$

24. [8] Find the **<u>magnitude</u>** of the **<u>resultant vector</u>** $\vec{u} + \vec{v}$ given that $|\vec{u}| = 2$, $|\vec{v}| = 3$ and that the angle between the vectors is 42°. Round to the **<u>nearest tenth</u>**.

25. [8] After taking off, an airplane reaches a speed of $120\ km/h$ on a course of 55°. A wind of $25\ km/h$ is blowing from 340°. What is the groundspeed of the plane?

26.[9] Find the **cube roots** of the complex number $-8i$. Write your answer in **standard notation,** $a+bi$.