# Chapter 1

# Sample Pretest

## Part I: SCIENTIFIC CALCULATOR REQUIRED

1. [6 points] Compute each number rounded to 3 decimal places. Please double check your answer.

   (a) $\dfrac{\sqrt{\sqrt[3]{2}+3}}{\pi}$ (b) $\sqrt{\dfrac{\pi^2+7}{1.3+\sqrt{7}}}$

   Answers

## Part II: NO CALCULATORS!

2. [3 points] Write each number as a decimal.

   (a) $2.1 \times 10^4$ (b) $3.26 \times 10^{-5}$ (c) $1 \times 10^{-2}$

   Answers

3. [3 points] Write each number in scientific notation.

   (a) 9582 (b) 1.245 (c) .000561

   Answers

4. [3 points] Compute the following. Simplify.

   (a) $-\dfrac{3}{4} - \dfrac{5}{12}$ (b) $\sqrt[3]{-|4^2 - 2^3|}$ (c) $\left(\dfrac{8}{27}\right)^{-2/3}$

   Answers

5. [10 points] Simplify each expression so that all the exponents are positive.

   (a) $h\dfrac{g^3h^{-2}}{q^2gh^3}$ (b) $\dfrac{a^{-1}+b^{-1}}{a^{-1}-b^{-1}}$

   Answers

6. [10 points] Rewrite each expression as a polynomial in standard form.

   (a) $(x-2)^2(x+2)$ (b) $2(x+7)-(2x-3)^2$

   Answers

7. [10 points] Factor over the reals.

   (a) $x^2-9x+14$ (b) $x^4-7x^2+12$

   Answers

8. [10 points] Perform the indicated operations and simplify the result as much as possible. (Assume $x \neq 1$ or 2.)

$$\frac{x+1}{x-1}+\frac{2x}{x-2}$$

   Answers

9. [15 points] Divide: $\dfrac{x^4-3x^3+2x^2-x+4}{x^2+3}$.

   Answers

10. [10 points] A right triangle has hypotenuse of length 10 feet and one leg in known to be 6 feet. What is the length of the other leg? What is the triangle's area?

    Answers

11. [10 points] An isosceles triangle has two legs of length 6 inches and one of length 4 inches. Find the triangle's area. Hint: draw a picture.

    Answers

12. [10 points] Construct a formula for the area $A$ of a circle in terms its circumference $C$. Hint: you should know the formulas for $A$ and $C$ in terms of the radius $R$.

    Answers

## 1.1 Answers

1. (a) 0.657 (b) 2.068

Return to Problem

2. (a) 21,000 (b) 0.0000326 (c) 0.01

Return to Problem

3. (a) $9.582 \times 10^3$ (b) 1.245 (c) $5.61 \times 10^{-4}$

Return to Problem

4. (a) $-\frac{7}{6}$ (b) $-2$ (c) $\frac{9}{4}$

Return to Problem

5. (a) $\frac{g^2}{q^2h^4}$ (b) $\frac{b+a}{b-a}$

Return to Problem

6. (a) $x^3 - 2x^2 - 4x + 8$ (b) $-4x^2 + 14x + 5$

Return to Problem

7. (a) $(x-7)(x-2)$ (b) $(x+\sqrt{3})(x-\sqrt{3})(x+2)(x-2)$

Return to Problem

8. $\frac{3x^2 - 3x - 2}{x^2 - 3x + 2}$

Return to Problem

9. Quotient $= x^2 - 3x - 1$. Remainder $= 8x + 7$.

Return to Problem

10. Leg = 8 feet. Area = 24 square feet.

Return to Problem

11. Area $= 4\sqrt{5}$ square inches.

Return to Problem

12. $A = \frac{C^2}{4\pi}$.

Return to Problem

# Chapter 2

# Practice Finals

No final exam can cover every single course objective. The practice finals given here are meant to give the student a general sense of the format and level of difficulty of a typical final exam. Studying these may be helpful but is in no way a substitute for studying your homeworks, class tests and quizzes. Just because a certain type of problem does not appear on either of these practice finals does not mean it will not be on your final exam.

## 2.1 Practice Final 1

1. [20 points; 4 points each]

   a) Simplify the following completely; express your answer using only positive exponents.

$$\left(\frac{27x^{-6}}{1000y^{-9}}\right)^{2/3}$$

   b) Factor over the reals: $2x^4 - 7x^2 - 4$.

   c) Solve for $x$: $\dfrac{3x-4}{x} = 2$.

   d) Solve for $x$: $\log_2 3x = 4$

   e) Find the equation of the line going through $(2, 3)$ and $(-1, 2)$, in slope-intercept form.

Answers

2. [20 points; 5 points each]

a) Find all real or complex values of $x$ that solve, $\dfrac{1-3x}{4} = \dfrac{3}{1+3x}$.

b) Find the equation, in standard form, of the circle passing through $(0, 1)$ with center $(2, -1)$.

c) Solve $|2x - 3| > 5$. Write your answer in interval notation.

d) Compute $\log_7 5$, rounded to 5 decimal places.

Answers

3. [20 points; 10 points each]

a) Let $g(x) = x^3 + 1$. Graph $y = g(x)$. b) Graph $y = g^{-1}(x)$ on the same grid. Label the intercepts on both graphs.

Answers

4. [20 points; 10 points each]

a) Sketch the graph of $y = \ln(x - 3)$. [6 points]
Label the intercepts and asymptotes. [4 points]

b) Graph $y = x^2 - 6x + 8$. [5 points]
Label all of the following: the intercepts, the vertex, and the axis of symmetry. [5 points]

Answers

5. [20 points; 10 points each]

a) List the potential rational zeros of $6x^2 + 7x - 3$, according to the Rational Zeros Theorem. [5 points] Factor $6x^2 + 7x - 3$. [5 points]

b) Factor $2x^3 + x^2 - 5x + 2$.

Answers

6. [20 points; 10 points each]

a) Solve for $x$ where $8^{2x-6} = 4^{x+1}$.

b) Solve for $x$ where $\log_3(x^2 + x) - \log_3(x^2 - x) = 1$.

Answers

7. [20 points] Let $f(x) = \dfrac{x}{x^2 - 4}$.

   a) [2 points] State the domain of $f$:

   b) [3 points] Is $f$ even, odd or neither?

   c) [3 points] Find all the asymptotes for the graph of $y = f(x)$.

   d) [2 points] Find all the intercepts for the graph of $y = f(x)$.

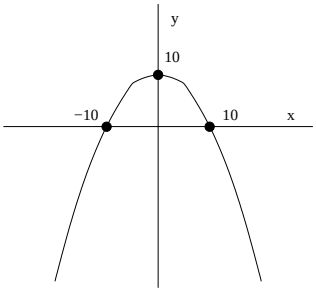
   e) [10 points] Sketch the graph **labeling** the intercepts and asymptotes.

Answers

8. [20 points; 10 points each]

   a) Find a 6 degree polynomial function $f(x)$ with real coefficients that has one zero at $x = 0$ with multiplicity 4, and a complex zero at $x = 2 - i$. Express your answer in standard form.

   b) Find the equation of the parabola below, expressed in standard form.

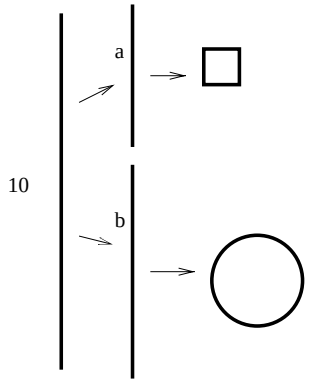



Answers

9. [20 points] Suppose you have 20 liters of a solution that is 30% isopropyl alcohol. How many liters of 80% alcohol would you have to add to get a solution that is 50% alcohol?

Answers

10. [20 points] A wire 10 meters long is to be cut into two pieces. Say the first piece has length $a$ and the second length $b$. Clearly $a + b = 10$. The first piece will be shaped into a square, the second into a circle.




a) [4 points] Find the area of the square as a function of $a$.

b) [6 points] Find the area of the circle as a function of $b$.

c) [2 points] Now find the area of the circle as a function of $a$; remember $a + b = 10$.

d) [4 points] Now find a formula for the total enclosed area in terms of $a$. Simplify it.

e) [4 points] This function should be a quadratic. For what value of $a$ is it a minimum?

Answers

## 2.2 Answers and Hints

1. [20 points; 4 points each]

   a) $\dfrac{9y^6}{100x^4}$

   b) $(2x^2+1)(x-2)(x+2)$

   c) $x = 4$.

   d) $x = 16/3$

   e) $y = x/3 + 7/3$.

Return to Problem

2. [20 points; 5 points each]

   a) $\pm i\sqrt{11}/3$.

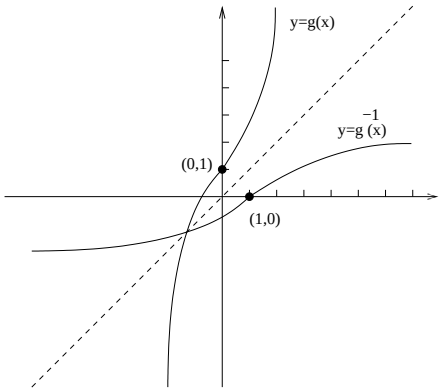
   b) $(x-2)^2 + (y+1)^2 = 8$.

   c) $(-\infty, -1) \cup (4, \infty)$

   d) 0.82709

Return to Problem

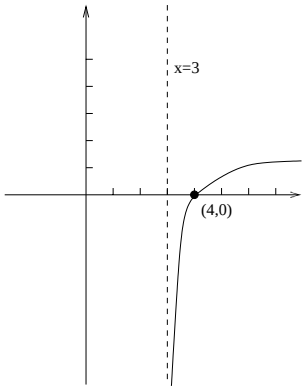
3. [20 points; 10 points each]



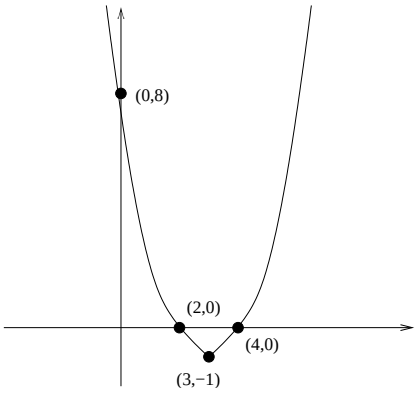

Return to Problem

4. [20 points; 10 points each]

a) $y = \ln(x-3)$




b) $y = x^2 - 6x + 8$.




Return to Problem

5. [20 points; 10 points each]

a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{6}$. $6x^2 + 7x - 3 = (2x+3)(3x-1)$ or $6(x + \frac{3}{2})(x - \frac{1}{3})$.

b) $2x^3 + x^2 - 5x + 2 = (2x-1)(x+2)(x-1)$.

Return to Problem

6. [20 points; 10 points each]

   a) $x = 5$

   b) $x = 2$. ($x = 0$ is invalid.)

Return to Problem

7. [20 points] Let $f(x) = \dfrac{x}{x^2 - 4}$.

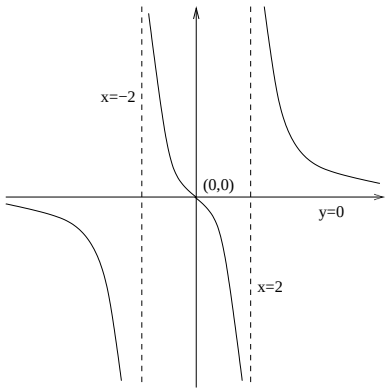
   a) [2 points] All real numbers except 2 and $-2$, or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

   b) [3 points] Odd.

   c) [3 points] Vertical: $x = 2$, $x = -2$. Horizontal: $y = 0$ (i.e., the $x$-axis).

   d) [2 points] $(0, 0)$ is the only intercept.

   e) [10 points] Sketch the graph labeling the intercepts and asymptotes.




Return to Problem

8. [20 points; 10 points each]

   a) $x^6 - 4x^5 + 5x^4$

   b) $-\frac{1}{10}x^2 + 10$.

Return to Problem

9. [20 points] Set up the equation $20(.3) + x(.8) = (20 + x)(.5)$. Thus, $x = 6\frac{2}{3}$.

Return to Problem

10. [20 points]

a) [4 points] $A_S = \dfrac{a^2}{16}$

b) [6 points] $A_C = \dfrac{b^2}{4\pi}$

c) [2 points] $A_C = \dfrac{(10-a)^2}{4\pi}$

d) [4 points] $A = A_S + A_C = \left(\dfrac{1}{16} + \dfrac{1}{4\pi}\right) a^2 - \dfrac{5}{\pi}a + \dfrac{25}{\pi}$

e) [4 points] $a_{\min} = \dfrac{40}{\pi + 4}$

Return to Problem

## 2.3 Practice Final 2

1. [20 points; 4 points each]

a) Simplify the following completely; express your answer using only positive exponents. (Assume both $x$ and $y$ are positive.)

$$\left(\frac{x^3 y^9}{8\sqrt{x^{12}y^{-6}}}\right)^{-1/3}$$

b) Solve for $t$ in the equation $A = A_0 e^{rt}$.

c) Rationalize the denominator of $\dfrac{x}{\sqrt{x} - \sqrt{2}}$.

d) A right triangle has area 3 and one leg of length 2. What is the length of its hypotenuse?

e) Multiply: $(5+3i)(2-i)$; express your answer in the form $a+bi$, where $a$ and $b$ are real numbers.

Answers

2. [20 points; 5 points each]

a) Find the equation of the line going through $(2, 3)$ and perpendicular to the line given by $y = 3x + 2$, in slope-intercept form.

b) Solve for $x$ in the equation $3^x = 11$. Round your answer to five decimal places.

c) Divide: $\dfrac{2+3i}{1-i}$; express your answer in the form $a + bi$, where $a$ and $b$ are real numbers.

d) Let $f(x) = 17 + \ln x^3$. Find $f^{-1}(20)$.

Answers

3. [20 points] The equation of a circle in general form is $x^2 + y^2 + 8x - 6y + 8 = 0$.

a) [10 points] Put the equation into standard form.

b) [4 points] What are the center and radius of the circle?

c) [6 points] Graph the circle.

Answers

4. [20 points; 10 points each]

a) Let $f(x) = x^3 + x^2 - 4x + 6$. Factor $f(x)$ over the complex numbers. Hint: check that $f(-3) = 0$.

b) Find a 4 degree polynomial function $f(x)$ with real coefficients that has complex zeros at $x = 3 - 2i$ and $x = 2 + i$. Express your answer in standard form.

Answers

5. [20 points; 10 points each]

a) Find all real values of $x$ such that $x^2 - x = 1$.

b) Find all real values of $x$ such that $e^{2x} - e^x = 1$.

Answers

6. [20 points; 10 points each]

   a) Solve the inequatity $\dfrac{x+1}{2x-3} \geq 0$; express your answer using interval notation.

   b) Solve the inequatity $\dfrac{x+1}{2x-3} \geq 1$; express your answer using interval notation.

Answers

7. [20 points] Let $f(x) = x^2 + 1$.

   a) [5 points] Graph $y = f(x)$.

   b) [10 points] Find a formula for the slope of the line segment joining $(2, f(2))$ with $(2+h, f(2+h))$ in terms of $h$; simplify it.

   c) [5 points] What value does the slope approach when $h$ tends toward 0?

Answers

8. [20 points] The weight of a colony of bacteria at time $t$ in hours obeys the equation $B(t) = B_0 e^{kt}$. The colony has an initial weight of 15 grams. In 10 hours the weight increased to 20 grams.

   a) [10 points] Find $k$. (Round your answer to six decimal places.)

   b) [10 points] How long will it take for the colony to double its weight? (Express your answer in hours and minutes, rounded to the nearest minute.)

Answers

9. [20 points] Let $f(x) = \dfrac{x+1}{x^2+2x-3}$.

   a) [2 points] State the domain of $f$:

   b) [3 points] Is $f$ even, odd or neither?

   c) [3 points] Find all the asymptotes for the graph of $y = f(x)$.

   d) [2 points] Find all the intercepts for the graph of $y = f(x)$.

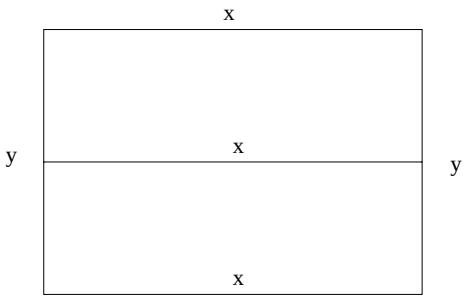
   e) [10 points] Sketch the graph **labeling** the intercepts and asymptotes.

Answers

10. [20 points; 10 points each] A gardener has 240 feet of fencing to encolse two adjacent rectangular growing areas as pictured. Both rectangles are to have the same dimensions.

    a) Express the total growing area as a function of $x$.

    b) What dimensions should be used so that the maximum growing area will be enclosed? (You must find both $x$ and $y$.)




Answers

## 2.4 Answers and Hints

1. [20 points]

    a) $\dfrac{2x}{y^4}$

    b) $t = \dfrac{\ln(A/A_0)}{r}$

    c) $\dfrac{x(\sqrt{x}+\sqrt{2})}{x^2-2}$

    d) $\sqrt{13}$

    e) $13+1i$, but $13+i$ is acceptable, even preferable.

Return to Problem

2. [20 points]

   a) $y = -\frac{1}{3}x + 3\frac{2}{3}$

   b) $x = \dfrac{\log 11}{\log 3} \approx 2.18266$

   c) $-\frac{1}{2} + \frac{5}{2}i$

   d) $f^{-1}(20) = e$.

Return to Problem

3. [20 points]

   a) $(x+4)^2 + (y-3)^2 = 17$

   b) Center $= (-4, 3)$. Radius $= \sqrt{17}$.

   c) Easy.

Return to Problem

4. [20 points]

   a) $(x+3)(x-(1-i))(x-(1+i))$

   b) $(x^2 - 6x + 13)(x^2 - 4x + 5) = x^4 - 10x^3 + 42x^2 - 82x + 65$

Return to Problem

5. [20 points]

   a) $\dfrac{1 \pm \sqrt{5}}{2}$

   b) $\ln\left(\dfrac{1+\sqrt{5}}{2}\right)$. Note that $\ln\left(\dfrac{1-\sqrt{5}}{2}\right)$ is undefined (or at least is not a real number) and hence is not a valid solution.

Return to Problem

6. [20 points]

   a) $(-\infty, -1] \cup (\frac{3}{2}, \infty)$

   b) $(\frac{3}{2}, 4]$

Return to Problem

7. [20 points]

   a) Easy.

   b) $m(h) = 4 + h$.

   c) $\lim_{h \to 0} m(h) = 4$.

Return to Problem

8. [20 points]

   a) $k \approx 0.028768$.

   b) 24 hours, 6 minutes.

Return to Problem

9. [20 points]

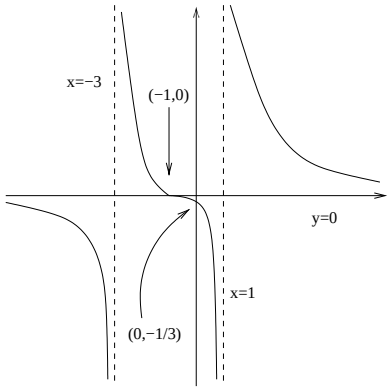
   a) All real numbers except $-3$ and 1, or $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.

   b) neither

   c) Vertical: $x = -3$ and $x = 1$. Horizontal: $y = 0$ (i.e., the $x$-axis).

   d) $(0, -\frac{1}{3})$, $(-1, 0)$.

   e)




Return to Problem

10. [20 points]

    a) $A = -3x^2/2 + 120x$.

    b) $x = 40$ and $y = 60$ feet.

Return to Problem