Part I. Problems in this section are mostly short answer and multiple choice. Little partial credit will be given. 5 points each.

1. Factor **completely**. Do not solve.

a) $2x^2+20x+18$

$2(x^2+10x+9)$

$\boxed{2(x+9)(x+1)}$

b) x^3-2x^2+3x-6

$x^2(x-2)+3(x-2)$

$\boxed{(x^2+3)(x-2)}$

2. Find the quotient and remainder.

$(x^3-3x+1)\div(x+2)$

Quotient: x^2-2x+1

Remainder: -1

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 1 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & -1 \end{array}$$

x^2-2x+1 ← Remainder

3. State the slope and y-intercept for the line $5x+2y=6$

$-5x \qquad -5x$

$\frac{2y}{2}=\frac{-5x+6}{2}$

$y=\frac{-5}{2}x+3$

$m=\frac{-5}{2}$ $\qquad$ $y-\text{int}:(0,3)$

(write it as an ordered pair)

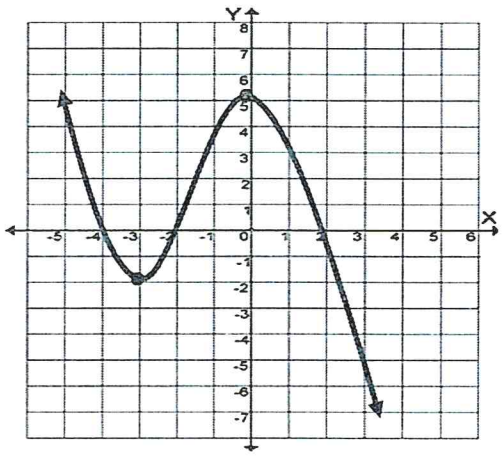
4. Given the graph of $f(x)$, state all x such that:

a) $f(x)$ is increasing (use interval notation)

$(-3, 0)$

b) $f(x) < 0$ (use interval notation)

$(-4, -2) \cup (2, \infty)$



5. Solve: $-3 \le \frac{2+x}{4} < 6$. Express your answer in interval notation.

$\cdot 4 \quad \cdot 4 \quad \cdot 4$

$-12 \le 2 + x < 24$

$-2 \quad -2 \quad -2$

$-14 \le x < 22$

$[-14, 22)$

6. Find the domain of the function $g(x) = \sqrt{x+3}$. Circle the correct answer.

a) $(-\infty, \infty)$
b) $(-3, \infty)$
c) $(-\infty, -3)$
d) $[-3, \infty)$
e) $(-\infty, -3]$

$x + 3 \ge 0$

$x \ge -3$

$[-3, \infty)$

7. Solve for x. Show all work and circle your final answer.

a) $x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x - 6 = 0 \quad x + 1 = 0$

$+6 \quad +6 \quad -1 \quad -1$

$x = 6 \quad x = -1$

b) $\cdot 4 \quad A = \frac{1}{4}xy \quad \cdot 4$

$4A = xy$

$\frac{4A}{y} = \frac{xy}{y}$

$\frac{4A}{y} = x \rightarrow x = \frac{4A}{y}$

8. Let $f(x)=5x-3$ and $g(x)=x^2+1$. Find and simplify.

a) $(f \circ g)(2) = f(g(2))$

$g(2)=(2)^2+1$
$=4+1$
$=5$

$= f(5)$
$= 5(5)-3$
$=25-3$
$=22$

b) $(f-g)(x) = f(x)-g(x)$
$=5x-3-(x^2+1)$
$=-x^2+5x-4$

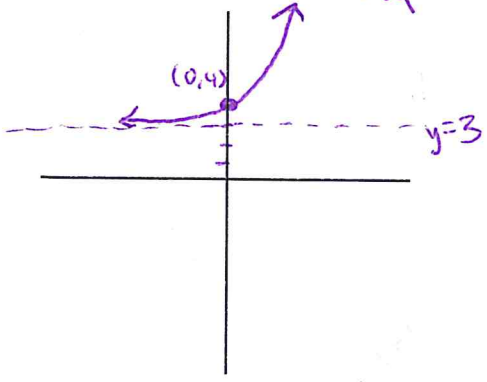
9. Solve ALGEBRICALLY. Show all work. $|2x-3|+4=11$

$-4 \quad -4$
$|2x-3|=7$

$2x-3=7$ $\quad$ $2x-3=-7$
$+3 \quad +3$ $\quad$ $+3 \quad +3$
$2x=10$ $\quad$ $2x=-4$
$\frac{2}{2} \quad \frac{2}{2}$ $\quad$ $\frac{2}{2} \quad \frac{2}{2}$
$x=5$ $\quad$ $x=-2$

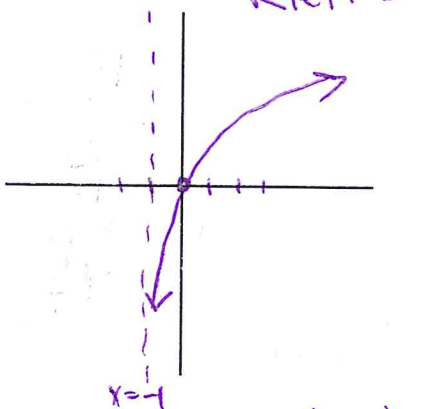
10. Graph each function. Dash in asymptotes where needed. Label all intercepts and asymptotes!

$f(x)=e^x+3$ ↑ up 3



Intercept in (x,y) form: $(0,4)$
Equation of asymptote: $y=3$

$g(x)=\ln(x+1)$ ← left 1



Intercept in (x,y) form: $(0,0)$
Equation of asymptote: $x=-1$

11. Find the product. Express in $a + bi$ form. $(4 + 6i)(1 - 3i)$

$4 - 12i + 6i - 18i^2$

$4 - 6i - 18(-1)$

$4 - 6i + 18$

$\boxed{22 - 6i}$

12. Find the slope of linear function f such that $f(3) = 2$ and $f(0) = -1$.

$(3, 2)$ $(0, -1)$

$m = \frac{-1-2}{0-3} = \frac{-3}{-3} = 1$

$m =$ 1

13. Write a polynomial of degree 3 that has zeros: 2 and $4i$. Write final answer in polynomial form (multiplied out).

zeros: $x = 2$ $x = 4i$ $x = -4i$

factors: $(x-2)(x-4i)(x+4i)$

$(x-2)(x^2 + 4ix - 4ix - 16i^2)$

$(x-2)(x^2 + 16)$

$x^3 - 2x^2 + 16x - 32$

$f(x) =$ $x^3 - 2x^2 + 16x - 32$

14. Given the point $(-2, 3)$, find a point that is symmetric to the given point:

a) with respect to the y-axis. $(-2,3)$ $(2, 3)$

b) with respect to the origin. $(2, -3)$

c) wrt x-axis

Part II. There are 9 problems in this section. Show all work. 10 points each.

15. A stone is thrown directly upward. Its height after t seconds is given by the function $h(t) = -3t^2 + 6t + 4$. The height of the stone is measured in feet. Show your work algebraically and include units on your answers.

a) What is the initial height of the stone?

$t = 0$

$h(0) = \boxed{4 \text{ ft.}}$

b) How long does it take for the stone to reach its maximum height?

time ↓ height ↓

$t = ?$ → vertex: $\left(\frac{-b}{2a}, h\left(\frac{-b}{2a}\right)\right)$

$t = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = \boxed{1 \text{ second.}}$

c) What is the maximum height the stone reaches?

vertex:

$h(1) = -3(1)^2 + 6(1) + 4$

$= -3 + 6 + 4$

$= \boxed{7 \text{ ft.}}$

16. Find all asymptotes, ~~x-intercepts~~ (zeros), and y-intercepts for the graph $f(x) = \frac{4x-6}{x-2}$.

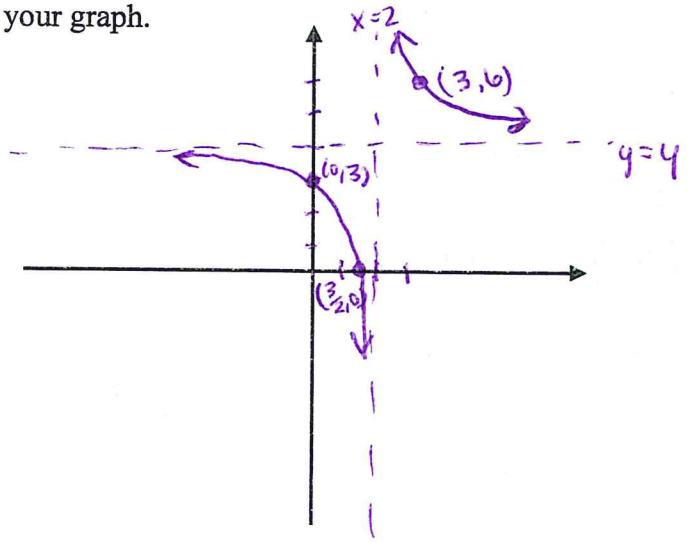
a) The equation of the vertical asymptote(s) is/are $x =$ 2.

b) The equation of the horizontal asymptote(s) is/are $y =$ 4.

c) The x-intercept (or zero) is at the point $\left(\frac{3}{2}, 0\right)$. $4x - 6 = 0 \Rightarrow 4x = 6 \Rightarrow x = \frac{6}{4} = \frac{3}{2}$

d) The y-intercept is at the point $(0, 3)$. $f(0) = \frac{4(0)-6}{0-2} = \frac{-6}{-2} = 3$

e) Sketch the graph of $f(x)$. Label all intercepts, asymptotes, and any additional points you found to help improve your graph.



x	y
3	$\frac{4(3)-6}{3-2} = \frac{12-6}{1} = 6$

17. Find a formula for the inverse given $f(x) = \frac{x+1}{3x-2}$.

$y = \frac{x+1}{3x-2}$

$(3y-2)x = \frac{y+1}{3y-2}(3y-2)$

$3xy - 2x = y + 1$

$-y \quad +2x \quad -y \quad +2x$

$3xy - y = 2x + 1 \longrightarrow y(3x-1) = 2x+1$

$\frac{y(3x-1)}{3x-1} = \frac{2x+1}{3x-1}$

$y = \frac{2x+1}{3x-1}$

$f^{-1}(x) = \frac{2x+1}{3x-1}$

18. Solve algebraically for x.

a) $4^{2x} = 8^{3x-1}$

$(2^2)^{2x} = (2^3)^{(3x-1)}$

$2^{4x} = 2^{9x-3}$

$4x = 9x - 3$

$-5x = -3$

$x = \frac{3}{5}$

b) $\ln(5x - 9) = 0$

$e^{\ln(5x-9)} = e^0$

$e^0 = 5x - 9$

$1 = 5x - 9$

$10 = 5x$

$\frac{10}{5} = \frac{5x}{5}$

$2 = x \rightarrow x = 2$

19. Suppose \$600 is invested in a savings account in which interest is compounded continuously at 2% per year. The amount of money in the account t years later is given by the equation: $A = 600e^{0.02t}$. Find the amount of time it would take the amount to reach \$2000. Leave your answer in exact form since no calculators are allowed.

$\frac{2000}{600} = \frac{600e^{0.02t}}{600}$

$\frac{10}{3} = e^{0.02t}$

$\ln\left(\frac{10}{3}\right) = \ln\left(e^{0.02t}\right)$

$\ln\left(\frac{10}{3}\right) = 0.02t$

$\frac{\ln\left(\frac{10}{3}\right)}{0.02} = t \longrightarrow t = \frac{\ln\left(\frac{10}{3}\right)}{0.02}$ years

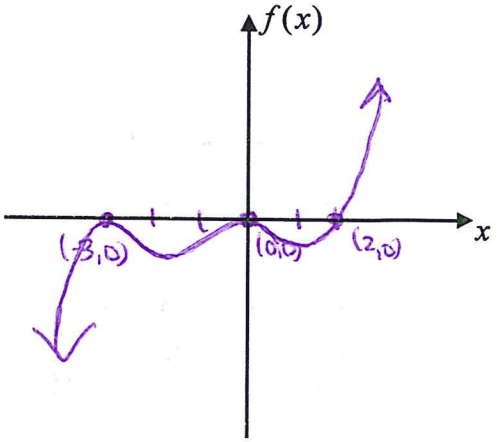
20. Given the function $f(x) = x^2(x-2)(x+3)^2$,

a) Find the y-intercept.

$f(0) = (0)^2(0-2)(0+3)^2$
$= 0 \cdot (-2)(9) = 0$ → (0, 0)

b) Find all zeros and state their multiplicities.

zero	multiplicity
0	2
2	1
-3	2



c) Is $f(x)$ tangent to the x-axis? Yes
If so, where?

at $x=0$ and $x=-3$

d) Draw ending behavior. $x^{(2)}(x^{(1)}-2)(x+3)^{(2)}$ → degree 5 ← odd
and positive leading coefficient

e) Sketch graph. Label all intercepts.

21. Solve algebraically: $(\sqrt{6x+7})^2 = (x+2)^2$ Check all solutions.

$6x+7 = (x+2)(x+2)$

$6x+7 = x^2+4x+4$
$-6x \quad -7 \qquad -6x \quad -7$

$0 = x^2-2x-3$

$0 = (x-3)(x+1)$

$x-3=0 \quad x+1=0$

$x=3 \quad x=-1$

Check:

$x=3$: $\sqrt{6(3)+7} = 3+2$
$\sqrt{25} = 5$ ✓

$x=-1$: $\sqrt{6(-1)+7} = -1+2$
$\sqrt{1} = 1$ ✓

22. Given the function $f(x) = x^2 + 6x + 5$

a) State the y-intercept.

$f(0) = 0^2 + 6(0) + 5 = 5$

$\boxed{(0, 5)}$

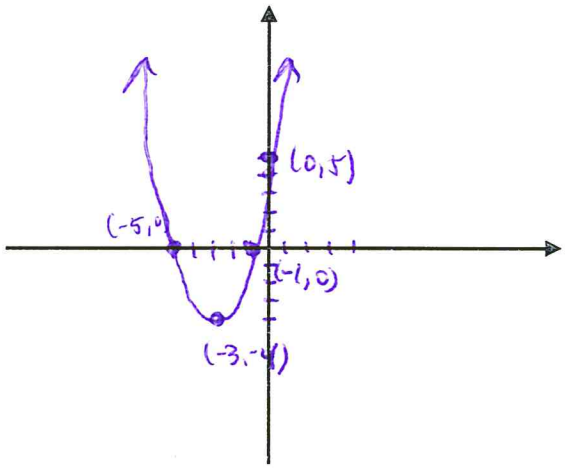
b) State the zeros of the function.

$x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$x + 5 = 0 \quad x + 1 = 0$

$\boxed{x = -5} \quad \boxed{x = -1}$



c) The vertex is (-3 , -4).

$\frac{-b}{2a} = \frac{-6}{2(1)} = -3$

$f(-3) = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4$

d) State the range of f.

$[-4, \infty)$

e) Graph. Label intercepts and vertex.

23. Given the polynomial $g(x) = x^3 - x^2 + x - 6$

a) State all possible rational zeros.

$1, 2, 3, 6$

$\div 1$

$\boxed{\pm\{1, 2, 3, 6\}}$

b) Find all zeros (real and complex.)

1 | 1 −1 1 −6
1 1 2
1 0 2 | −4

not zero

−1 | 1 −1 1 −6
−1 2 −3
1 −2 3 | −9

2 | 1 −1 1 −6
2 2 6
1 1 3 | 0 ← zero, so $x = 2$ is a zero

$x^2 + x + 3 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$

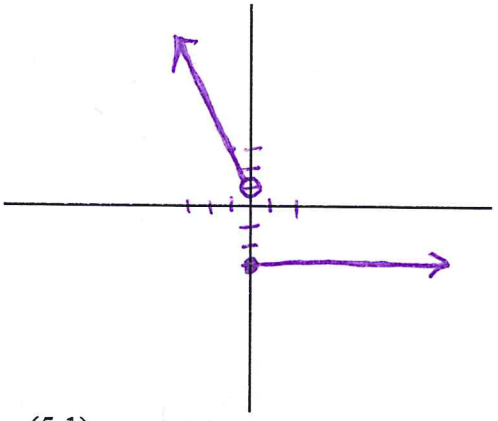
Answer: $x = 2 \quad x = \frac{-1 + i\sqrt{11}}{2} \quad x = \frac{-1 - i\sqrt{11}}{2}$

Part III. There are 6 problems in this section. Choose any 4. Indicate in the boxes the problems you want graded. 10 points each. If you do not indicate which 4, the first 4 will be graded. No Extra Credit!

☐ Grade

24. Graph the following function.

$$f(x)=\begin{cases}-2x+1, & x<0\\ -3, & x\geq 0\end{cases}$$



25. The graph to the right is a circle with center (5,1).

☐ Grade

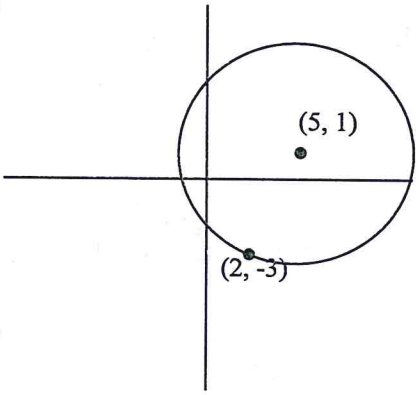
a) Find the length of the radius.

r = distance between (5,1) & (2,-3)

$= \sqrt{(5-2)^2+(1-(-3))^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{9+16} = \sqrt{25} = \boxed{5}$



b) State the equation of the circle in standard form.

$\boxed{(x-5)^2+(y-1)^2=25}$

☐ Grade

26. Given $f(x) = 2x^2 + 4x$, find and simplify $\dfrac{f(x+h)-f(x)}{h}$.

note: $f(x+h) = 2(x+h)^2+4(x+h)$

$= 2(x^2+2xh+h^2)+4x+4h$

$= 2x^2+4xh+2h^2+4x+4h$

$$\frac{f(x+h)-f(x)}{h} = \frac{2x^2+4xh+2h^2+4x+4h-(2x^2+4x)}{h}$$

$$= \frac{h(4x+2h+4)}{h} = \boxed{4x+2h+4}$$

27. Solve algebraically for x. $\log_2 x + \log_2(x-2) = 3$

❐ Grade

$\log_2(x(x-2)) = 3$

$2^3 = x^2 - 2x$

$8 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2) \Rightarrow$ $x = 4$ $x \neq -2$

Check:

$x=4$ $\log_2 4 + \log_2(2) = 3$

$2 + 1 = 3$ ✓

$x=-2$ $\log_2(-2)$ ↑ can't do!

❐ Grade

28. Solve $\frac{x+2}{x-3} \geq 0$. Express in interval form. To receive full credit you must show work that supports your answer.

VA: $x-3=0$

$x=3$

zeros: $x+2=0$

$x=-2$

$(-\infty,-2]$ $[-2,3)$ $(3,\infty)$

-2 3

$x=-3$ $x=0$ $x=4$

Test values:

$x=-3$ $\frac{-3+2}{-3-3} = \frac{-1}{-6} = \frac{1}{6} \geq 0$ ✓ True!

$x=0$ $\frac{0+2}{0-3} = -\frac{2}{3} \geq 0$ ✗ False!

$x=4$ $\frac{4+2}{4-3} = \frac{6}{1} = 6 \geq 0$ ✓ True

$(-\infty,-2] \cup (3,\infty)$

❐ Grade

29. Consider the following system: $\begin{cases} 2x - 3y = -6 \\ x + y = 2 \end{cases}$

a) Solve algebraically
Show all your work.

$2x-3y=-6$

$3(x+y=2) \rightarrow 3x+3y=6$

$5x=0$

$x=0$

$y=2$

b) Solve graphically and explain how you obtained your answer by looking at the graph.

$2x-3y=-6$

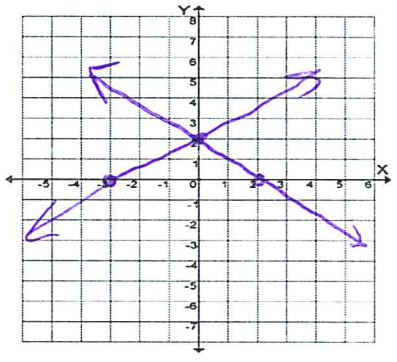
x-int: $(-3,0)$

y-int: $(0,2)$

$x+y=2$

x-int: $(2,0)$

y-int: $(0,2)$



$(0,2)$ point where the graphs intersect

Answer: $(0,2)$