Sample Final Exam 1 For MATH 150

You are responsible for all material covered in the course syllabus. These Sample Finals are meant to give students a general feel for the length, scope and difficultly of a Final Exam. The best way to use these Sample Finals is to study all the course material, including homeworks, worksheets, quizzes, tests and the textbook thoroughly. Then you can take a Sample Final in two hours without using the textbook, notes or any thing not permitted during the actual final. Then compare your answers with those provided. Then find your mistakes and go back and study the sections of the textbook you are still weak on. Then repeat.

Study hard and good luck!

1. (20 points) Let $f(x) = \dfrac{3x^2 + 15x + 18}{x^2 + x - 2}$. Find the following limits. Justify your answers. Hint: Factor!

   a. $\lim_{x \to -2} f(x)$ b. $\lim_{x \to 1^-} f(x)$

   c. $\lim_{x \to 1^+} f(x)$ d. $\lim_{x \to \infty} f(x)$

2. (10 points) Find a number c such that the function $f(x)$ defined below is always continuous.

$$f(x) = \begin{cases} x^2 - c & \text{if} \quad x < 3, \\ cx + 10 & \text{if} \quad x \geq 3. \end{cases}$$

3. (10 points) Let $g(x) = \frac{\sin 3x}{5x}$. Find the following limits. Justify your answers.

   a. $\lim_{x \to 0} g(x)$ b. $\lim_{x \to \infty} g(x)$

4. (10 points) Let $f(x) = \sqrt{2x+1}$. Use the definition of the derivative and the properties of limits to find $f'(x)$. Show all work.

5. (20 points) Find the derivatives of the functions below. You do not need to simplify your answers.

   a. $f(x) = \frac{\sec x}{2+\cot x}$

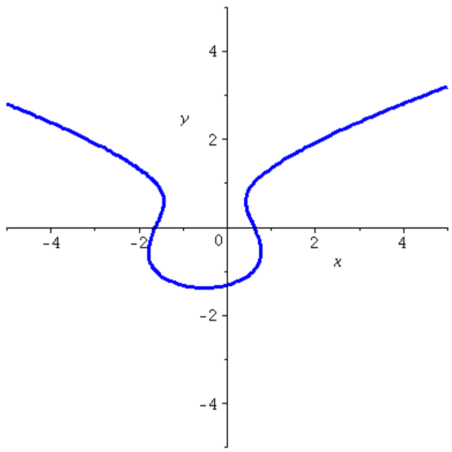
   b. $g(x) = \pi^2 + xe^{2x} + \ln x^2$

   c. $h(x) = x^x$

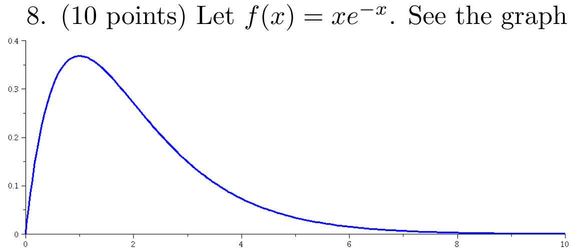
   d. $F(x) = \int_7^{x^2} \sin(t^3)\, dt$

6. (10 points) A particle moves in a straight line and has acceleration function $a(t) = 1 - 6t + 48t^2$. If the initial velocity is 2 ft/sec and the initial position is 1 foot, find its position function $s(t)$.

7. (10 points) The relation $y^3 - y = x^2 + x - 1$ is graphed below. It appears that there is one point where the tangent line is horizontal. Find the x coordinate of this point.



8. (10 points) Let $f(x) = xe^{-x}$. See the graph below.

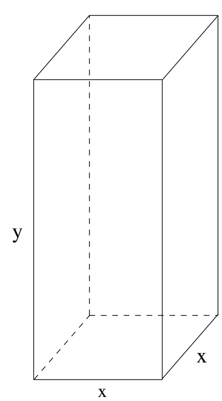


a. Use calculus to find the coordinates where $f(x)$ has its maximum value. Mark this point on the graph.

b. Use calculus to find the coordinates for the inflection point of $f(x)$. Mark this point on the graph.

9. (10 points) Two cars start moving from the same point at the same time. One travels south at 40 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing four hours later?

10. (15 points) Consider a box with a square base. If the surface area is 100 square ft, what dimensions maximize the volume?



11. (20 points.) Let $f(x) = e^x - 3e^{-x} - 4x$.

a. Find $f'(x)$. Where is $f'(x) = 0$?

b. Find any intervals where f is increasing.

c. Find any intervals where f is decreasing.

d. Find the local extrema of f. Give exact values and numerical approximations to three decimal places.

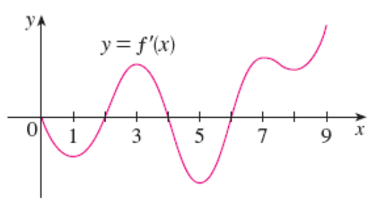
e. Find the inflection point of f. Give exact values and numerical approximations to three decimal places.

f. Find any intervals where f is concave up.

g. Find any intervals where f is concave down.

h. Sketch a rough graph of $y = f(x)$ over x in $[-1, 3]$. Mark the local extrema and the inflection point.

12. (15 points) Below is the graph of $f'(x)$, the **derivative** of a function $f(x)$. (a - e are three points each.)



a. On which intervals if any is the function $f(x)$ increasing?

b. On which intervals if any is the function $f(x)$ decreasing?

c. For which values of x does $f(x)$ have a relative maximum?

d. For which values of x does $f(x)$ have a relative minimum?

e. Make a very rough sketch of the graph of $f(x)$, assuming $f(0) = 0$.

13. (20 points) Do the following indefinite integrals.

a. $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}\,dx$ b. $\displaystyle\int \tan t + \tfrac{1}{t+1} + e^t\,dt$

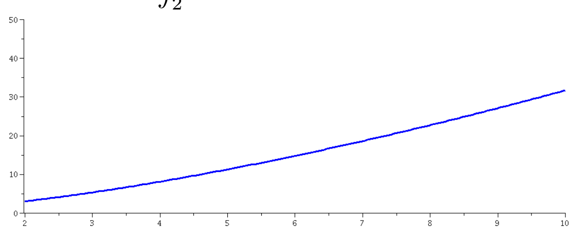
c. $\displaystyle\int \frac{\sin 2\theta}{1+\sin^2\theta}\,d\theta$ d. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

14. (10 points) Evaluate the following definite integrals.

a. $\displaystyle\int_{-1}^{1} \tan^3 x^5\,dx$ b. $\displaystyle\int_0^{\sqrt{\pi}} x\cos x^2\,dx$

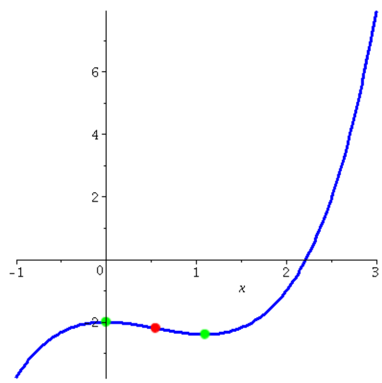
15. (10 points) Use the Midpoint Rule with $n = 4$ to approximate the integral below to four decimal places.

$\displaystyle\int_2^{10} \sqrt{x^3+1}\,dx.$

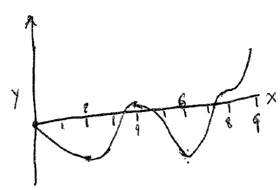


Answers

1. a. -1 b. $-\infty$ c. ∞ d. 3 (The hint about factoring only applies to a-c. For d, if you have seen L'Hospotal's Rule you could use it, but it is faster to use the methods of Section 1.6. See Example 5 in that section.)
2. $c = -1/2$ Compare to Exercises 31 and 32 in Section 1.5.
3. a. $3/5$ Use methods of Section 1.4 or 3.7. b. 0
4. See Exercise 21, Section 2.2.
5. $s(t) = 1 + 2t + \frac{1}{2}t^2 - t^3 + 4t^4$.
6. a. $\frac{\sec x \tan x(2+\cot x)+\sec x \csc^2 x}{(2+\cot x)^2}$ b. $e^{2x} + 2xe^{2x} + \frac{2}{x}$ c. $x^x(1 + \ln x)$ (Use logarithmic differentiation; see Section 3.3.) d. $2x\sin(x^6)$ (See Section 5.4.)
7. $-1/2$ (Use implicit differentiation to find y', then set $y' = 0$ and solve for x.)
8. Max is at $(1, 1/e) \approx (1.0000, 0.3679)$. Infection point is $(2, 2/e^2) \approx (2.0000, 0.2707)$.
9. 50 mi/h
10. The base edge length and the height are both $\sqrt{\frac{50}{3}}$, or about 4 ft and 1 in. So, it is a cube!
11. a. $f'(x) = e^x - 3e^{-x} - 4x$. 0 and $\ln 3 \approx 1.009$. b. $(-\infty, 0) \cup (\ln 3, \infty)$ c. $(0, \ln 3)$ d. local max: -2, local max: $2 - \ln 81 \approx -2.394$. e. $(\frac{1}{2}\ln 3, -\ln 9) \approx (0.549, -2.197)$. f. $(-\infty, \frac{1}{2}\ln 3)$ g. $(\frac{1}{2}\ln 3, \infty)$ h. See below.



12. a. $(2,4)$, $(6,9)$ b. $(0,2)$, $(4,6)$ c. 2, 6 d. 0, 4 e. See graph below.



13. a. $-\frac{1}{4}\sqrt{1-4x^2} + C$ (See Example 3, Section 5.5) b. $\ln|\sec t| + \ln|t| + e^t + C$ c. $\ln(1 + \sin^2\theta) + C$ (Compare to Exercise 31 in Section 5.5.) d. $\frac{2}{3}(1 + e^x)^{\frac{3}{2}} + C$
14. a. 0 (Use symmetry of odd functions.) b. 0
15. $2\left(\sqrt{28} + \sqrt{126} + \sqrt{344} + \sqrt{730}\right) \approx 124.1644$