MATH 150 SKILLS ASSESSMENT

The purpose of this test is purely diagnostic (before beginning your review, it will be helpful to assess both strengths and weaknesses). All of the test problems are essential to first semester calculus. Answers are provided, and each answer has references to the relevant review topics. If anything is unclear, the review material should help.

You may click on the blue words if you wish to jump to an answer or the review topics.

If you would like to print the printer friendly version of the Skills Assessment so you can work it out on paper, please click Print. (The entire Skills Assessment file is too large to print.)

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Course Review Home Math Home

[1] Sketch a graph – indicate domain and range of f.

a) $f(x) = x^2 + 2$ Answer

b) $f(x) = |x - 2|$ Answer

c) $f(x) = -\ln x$ Answer

[2] Given $f(x) = \begin{cases} x^2, & -3 \leq x < 0 \\ x+1, & 0 \leq x < 2 \\ 4, & 2 \leq x \leq 5 \end{cases}$

a) Find $f(-3)$, $f(0)$, $f(2)$, $f(e)$. Answer

b) Sketch a graph of f. Answer

[3] a) For the function $f(x) = x^2 - 2x$, find and simplify $f(x+h) - f(x)$. Answer

b) Let $f(x) = 2x + 1$, $g(x) = \sqrt{x}$, $h(x) = \sin x$. Find the following compositions:

i) $f(g(x))$ Answer

ii) $h(g(x))$ Answer

iii) $g(h(f(x)))$ Answer

c) Find and simplify the difference quotient $\dfrac{f(x) - f(3)}{x - 3}$ if $f(x) = \dfrac{1}{x}$. Answer

d) Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ given $f(x) = \sqrt{x+2}$. Rationalize the numerator. Answer

[4] a) Simplify $\dfrac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$. Answer

b) Change $x^{1/3}+(x-1)\cdot\dfrac{1}{3}x^{-2/3}$ to an equivalent form whose denominator is $3x^{2/3}$. Answer

c) Given $f(x)=\dfrac{-5x(x-4)}{2\sqrt{5-x}}$;

i. find the domain of f. Answer

ii. find the intervals where $f>0$ and $f<0$. Answer

[5] a) Given $y = \ln(x\sqrt{x^2+1})$, convert into an expression involving sums, differences, and multiples of logarithms. Answer

b) Simplify: $e^{-2\ln x}$ Answer

c) Solve for x:

i. $\ln(x-1) = 2$. Answer

ii. $e^{2x} - 2xe^{2x} = 0$ Answer

[6] a) Given $g(x) = \dfrac{2x-3}{x^2+1}$,

i. state the domain of g; Answer

ii. find $g(0)$; Answer

iii. find all x such that $g(x) = 0$; Answer

iv. find all asymptotes of g. Answer

b) Sketch a graph of $f(x) = (x^2-3)(5-x)^2$; include all intercepts. Answer

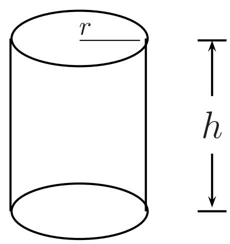
c) Sketch a graph of $f(x) = \dfrac{x+3}{3-2x}$; include asymptotes and all intercepts. Answer

[7] a) The position of an object above the ground is given by $s(t) = 128t - 16t^2$, with s measured in feet and t in seconds.

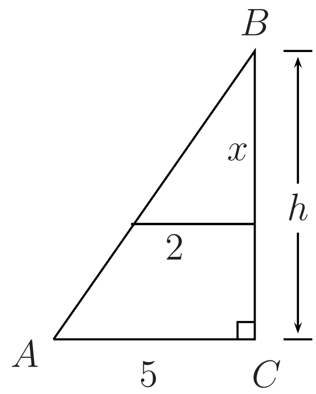
i. Find the maximum height reached by the object. Answer

ii. How long does it take for the object to return to the ground? Answer

b) A right circular cylinder with radius r and height h has a volume of 5 cm^3. Express its surface area as a function of r, given $SA = 2\pi r^2 + 2\pi rh$. Answer



[7] c) Express the area of ΔABC as a function of x. Answer



B R E A K   T I M E !

Hey, you've been working hard. Now it's time for a break. Kick back and relax a little. Grab some munchies. How about some brownies? We have a recipe for brownies that we guarantee is the best ever.

Math Brownie Recipe 1

O.K., you're relaxed and refreshed. It's time to get back to work.

Continued on next page ...

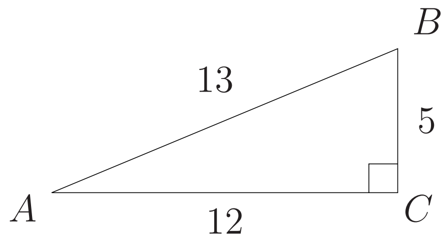
8. a) $360^\circ = \_\_\_\_\_\_\_\_$ radians. Answer

b) $\_\_\_\_\_\_\_\_^\circ = \frac{5\pi}{6}$ radians. Answer

c) $225^\circ = \_\_\_\_\_\_\_\_$ radians. Answer

9. For each figure below, find exact values for the quantities indicated.

a)

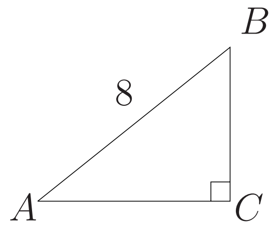


$\sin A =$ ______; $\sin B =$ ______; $\cos A =$ ______;

$\tan A =$ ______; $\sec B =$ ______; $\cot A =$ ______.

Answer

9. b)



If angle $A = \dfrac{\pi}{6}$, then side $BC =$______.

If angle $A = \dfrac{\pi}{4}$, then side $AC =$______.

Answer

10. Consider a circle of radius 2 centered at the origin. A central angle $\theta = \dfrac{\pi}{3}$ radians will subtend ("mark off") an arc on the circle of length ________. Answer

11. a) Suppose $0 < \theta < \dfrac{\pi}{2}$. If $\sin\theta = x$, find $\cos\theta$ and $\tan\theta$ in terms of x. Answer

b) In which quadrant does θ terminate if $\cos\theta < 0$ and $\tan\theta < 0$? Answer

12. Without the use of a calculator, find the exact value of each expression below.

a) $\sin \dfrac{7\pi}{6}$ Answer

b) $\cos \left(\dfrac{-\pi}{2} \right)$ Answer

c) $\tan \dfrac{7\pi}{4}$ Answer

d) $\sec \dfrac{13\pi}{6}$ Answer

e) $\sin \left(\dfrac{-15\pi}{4} \right)$ Answer

13. Without using a claculator, complete the table below giving exact values.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$											
$\cos\theta$											
$\tan\theta$											

Answer

14. By memory, graph $y = \cos x$, $-2\pi \leq x \leq 2\pi$. Label all intercepts, maxima, and minima. Answer

15. Verify the following trigonometric identities.

a) $\dfrac{1+\sec\theta}{\csc\theta} = \sin\theta + \tan\theta$ Answer

b) $(1-\cos^2\theta)(1+\cot^2\theta) = 1$ Answer

c) $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta$ Answer

16. Evaluate the following without a calculator.

a) $\arcsin(1)$ or $\sin^{-1}(1)$ Answer

b) $\arccos(0)$ Answer

c) $\arcsin\left(-\frac{1}{2}\right)$ Answer

d) $\tan(\arcsin(x))$, where $0 < x < 1$ Answer

17. Without using a calculator, graph $f(x) = \arctan x$ (or $f(x) = \tan^{-1} x$).

Domain $f =$ ____________.

Range $f =$ ____________.

Answer

18. Solve the following trigonometric equations.

a) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ Answer

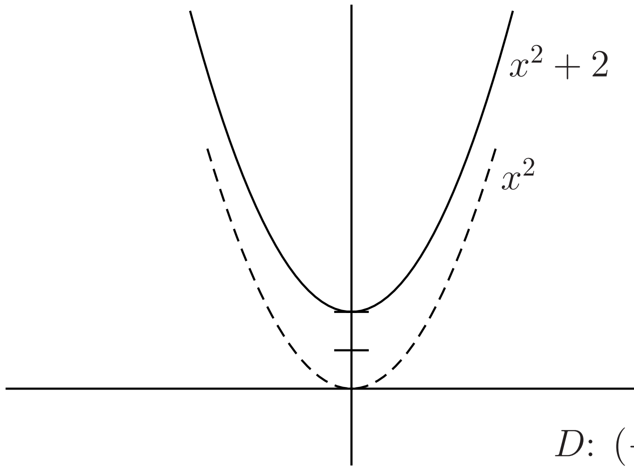
b) $2\sin^2\theta - \sin\theta - 1 = 0,\ 0 \leq \theta < 2\pi$ Answer

c) $\sin\theta + \sin 2\theta = 0,\ 0 \leq \theta < 2\pi$ Answer

d) $\sin\theta - \cos\theta = 0,\ 0 \leq \theta < 2\pi$ Answer

ANSWERS to MATH 150 SKILLS ASSESSMENT

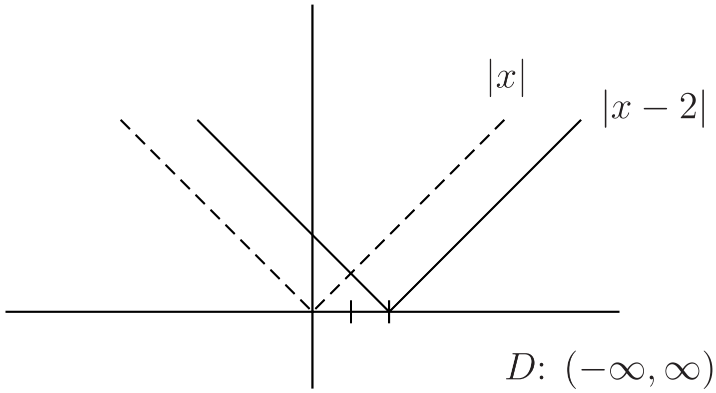
1. a)



D: $(-\infty, \infty)$
R: $[2, \infty)$

Return to Problem Review Topic 1

1. b)

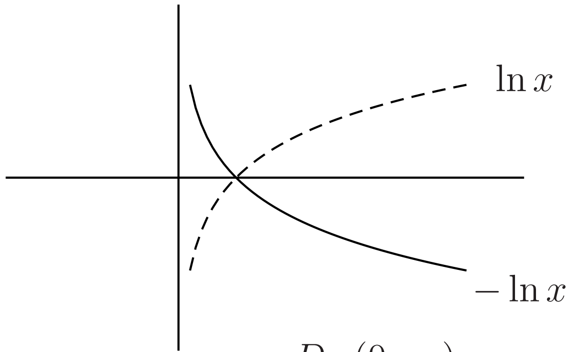


D: $(-\infty, \infty)$

R: $[0, \infty)$

Return to Problem Review Topic 1

1. c)



D: $(0, \infty)$

R: $(-\infty, \infty)$

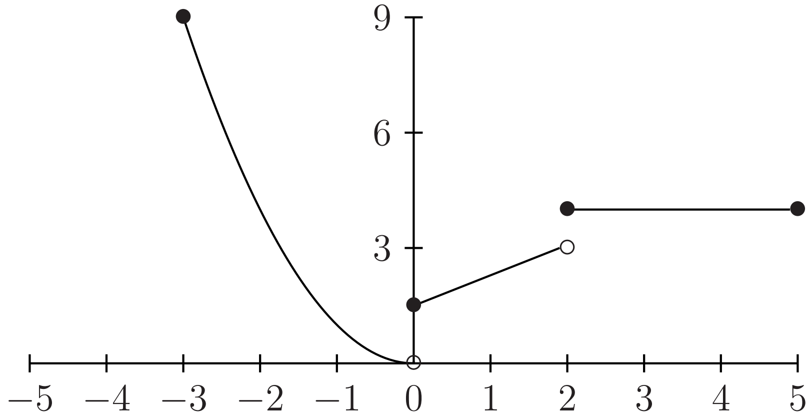
Return to Problem Review Topic 1

2. a)

$$f(-3) = 9$$
$$f(0) = 1$$
$$f(2) = 4$$
$$f(e) = 4$$

Return to Problem Review Topic 2

2. b)



Return to Problem Review Topic 2

3. a) $$\begin{aligned} f(x+h)-f(x) &= (x+h)^2-2(x+h)-(x^2-2x) \\ &= 2xh+h^2-2h \end{aligned}$$

Return to Problem Review Topic 3

3. b) i. $2\sqrt{x} + 1$

Return to Problem Review Topic 3

3. b) ii. $\sin\sqrt{x}$

Return to Problem Review Topic 3

3. b) iii. $\sqrt{\sin(2x+1)}$

Return to Problem Review Topic 3

3. c) $$\frac{f(x)-f(3)}{x-3} = \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$$
$$= \frac{3-x}{3x}\cdot\frac{1}{x-3}$$
$$= -\frac{1}{3x}$$

Return to Problem Review Topic 3

3. d)

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\\ &= \frac{(\sqrt{x+h+2})^2-(\sqrt{x+2})^2}{h(\sqrt{x+h+2}+\sqrt{x+2})}\\ &= \frac{h}{h(\sqrt{x+h+2}+\sqrt{x+2})}\\ &= \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}\end{aligned}$$

Return to Problem Review Topic 3

4. a)

$$\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2} = \frac{-2x[(1+x^2)+(1-x^2)]}{(1+x^2)^2}$$

$$= \frac{-2x(2)}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$

Return to Problem Review Topic 4

4. b)

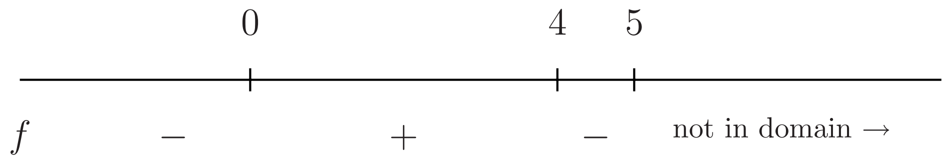
$$x^{1/3} + (x-1)\cdot\frac{1}{3}x^{-2/3} = \frac{3x^{2/3}\left[x^{1/3} + (x-1)\cdot\frac{1}{3}x^{-2/3}\right]}{3x^{2/3}}$$

$$= \frac{3x + (x-1)x^0}{3x^{2/3}}$$

$$= \frac{4x-1}{3x^{2/3}}$$

Return to Problem Review Topic 4

4. c) Domain: $(-\infty, 5)$, critical values: 0, 4, 5



$$f > 0 \text{ on } (0,4), \qquad f < 0 \text{ on } (-\infty, 0) \cup (4,5)$$

Return to Problem Review Topic 4

5. a) $y = \ln x + \dfrac{1}{2}\ln(x^2+1)$

Return to Problem Review Topic 5

5. b) $e^{\ln x^{-2}} = x^{-2}$

Return to Problem Review Topic 5

5. c) i.

$$\begin{aligned} e^{\ln(x-1)} &= e^2 \\ x - 1 &= e^2 \\ x &= e^2 + 1 \end{aligned}$$

Return to Problem Review Topic 5

5. c) ii. $e^{2x}(1-2x)=0$

$e^{2x} \neq 0$ for any x

$1-2x=0$ at $x=\dfrac{1}{2}$

Return to Problem Review Topic 5

6. a) i. Domain of g: $(-\infty, \infty)$

Return to Problem Review Topic 6

6. a) ii. $g(0) = -3$

Return to Problem Review Topic 6

6. a) iii. $g(x) = 0$ when $2x - 3 = 0$. $x = \dfrac{3}{2}$.

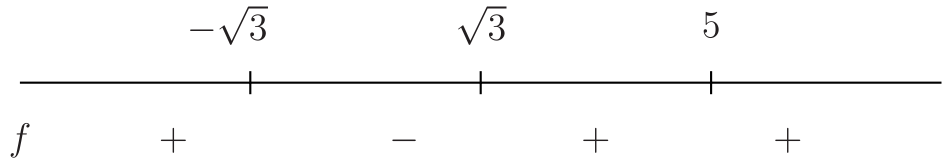
Return to Problem Review Topic 6

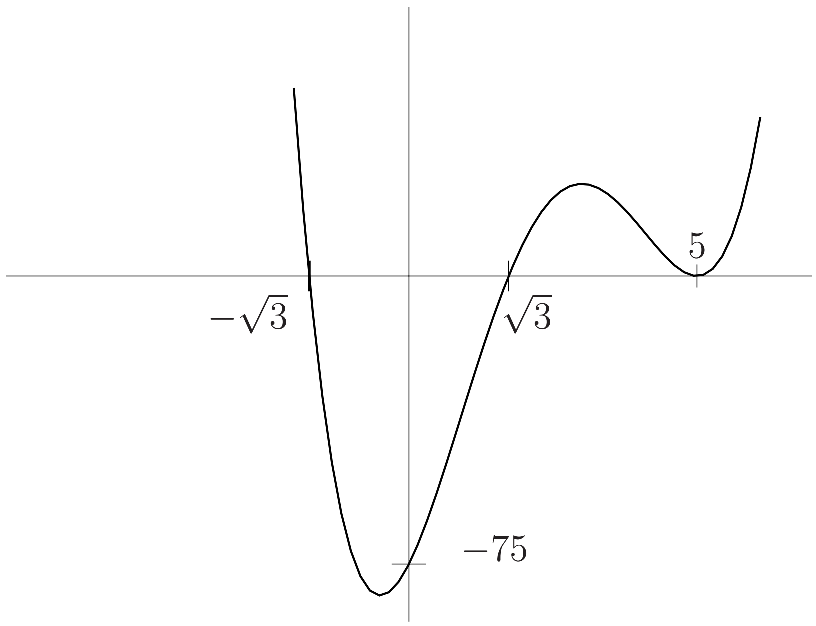
6. a) iv. HA $y = 0$

VA None, $x^2 + 1 \neq 0$ for all real x.

Return to Problem Review Topic 6

6. b) Critical points: $\pm\sqrt{3}$, 5, $f(0) = -75$





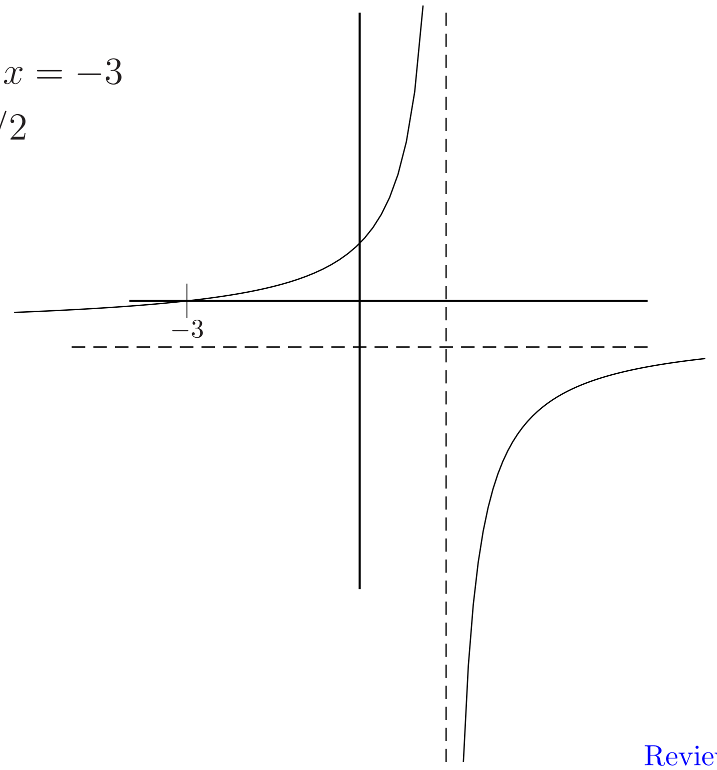
Return to Problem Review Topic 6

6. c) $f(0) = 1$

$f(x) = 0$ at $x = -3$

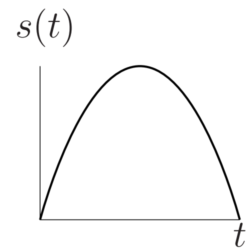
HA $y = -1/2$

VA $x = 3/2$



Return to Problem Review Topic 6

7. a) i) Finding the max or min of a quadratic function is the same as finding its vertex (calculus uses other methods).



The maximum value of t occurs at

$$\begin{aligned} t &= -\frac{b}{2a} \\ &= \frac{-128}{-32} \\ &= 4 \text{ sec} \end{aligned}$$

It takes 4 sec. to reach a height of $s(4)$ or 256 ft.

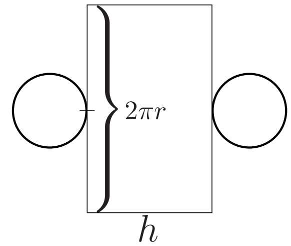
Return to Problem Review Topic 7

7. a) ii)

$$\begin{aligned} s = 128t - 16t^2 &= 0 \\ 16t(8-t) &= 0 \\ t = 0,\ t &= 8 \end{aligned}$$

It takes 8 sec. to return to the ground.

Return to Problem Review Topic 7



7. b) Unrolled, a cylinder looks like this:
Its surface area consists of two circles and a rectangle

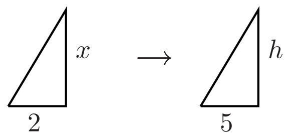
$$SA = 2\pi r^2 + 2\pi r h$$

Since $V = \pi r^2 h = 5$ becomes $h = \dfrac{5}{\pi r^2}$, substitution yields

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi r \left(\frac{5}{\pi r^2}\right) \\ &= 2\pi r^2 + \frac{10}{r}\,. \end{aligned}$$

Return to Problem Review Topic 7

7. c) Using similar triangles,



$$\frac{x}{2} = \frac{h}{5}$$
$$h = \frac{5}{2}x$$

$$\begin{aligned} A &= \frac{1}{2} \cdot 5 \cdot h \\ &= \frac{1}{2} \cdot 5 \left(\frac{5}{2}x \right) \\ A(x) &= \frac{25}{4}x \end{aligned}$$

Return to Problem Review Topic 7

8. a) 2π

Return to Problem Review Topic 8

8. b) 150°

Return to Problem Review Topic 8

8. c) $\dfrac{5\pi}{4}$ radians

Return to Problem Review Topic 8

9. a) $\sin A = \dfrac{5}{13}\,;$ $\sin B = \dfrac{12}{13}\,;$ $\cos A = \dfrac{12}{13}\,;$

$$\tan A = \frac{5}{12}\,; \qquad \sec B = \frac{13}{5}\,; \qquad \cot A = \frac{12}{5}\,.$$

Return to Problem Review Topic 9a

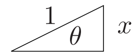
9. b) If angle $A = \dfrac{\pi}{6}$, then $\sin\dfrac{\pi}{6} = \dfrac{1}{2} = \dfrac{BC}{8}$, or $BC = 4$.

If angle $A = \dfrac{\pi}{4}$, then $\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} = \dfrac{AC}{8}$, or $AC = 4\sqrt{2}$.

Return to Problem Review Topic 9b

10. Using the arc length formula $s = r\theta$, we get $s = \dfrac{2\pi}{3}$.

Return to Problem Review Topic 10

11. a) If $\sin\theta = x = \dfrac{x}{1}$, we can write 

. Thus $\cos\theta = \dfrac{\sqrt{1-x^2}}{1}$ and $\tan\theta = \dfrac{x}{\sqrt{1-x^2}}$.

Return to Problem Review Topic 9c

11. b) Second quadrant.

Return to Problem Review Topic 9d

12. a) $\sin\dfrac{7\pi}{6} = -\dfrac{1}{2}$

Return to Problem Review Topic 9d

12. b) Since $\cos(-x) = \cos x$, $\cos\left(-\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$.

Return to Problem Review Topic 12

12. c) $\tan\dfrac{7\pi}{4} = \dfrac{\sin\dfrac{7\pi}{4}}{\cos\dfrac{7\pi}{4}} = \dfrac{-\dfrac{\sqrt{2}}{2}}{\dfrac{\sqrt{2}}{2}} = -1$

Return to Problem Review Topic 12

12. d)
$$\sec\frac{13\pi}{6} = \frac{1}{\cos\left(\frac{13\pi}{6}\right)} = \frac{1}{\cos\left(\frac{\pi}{6}+2\pi\right)} = \frac{1}{\cos\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

Return to Problem Review Topic 11

12. e) Since $\sin(-x) = -\sin x$,

$$\sin\left(-\frac{15\pi}{4}\right) = -\sin\left(\frac{15\pi}{4}\right)$$
$$= -\sin\left(\frac{7\pi}{4} + 2\pi\right) = -\sin\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

Return to Problem Review Topic 11

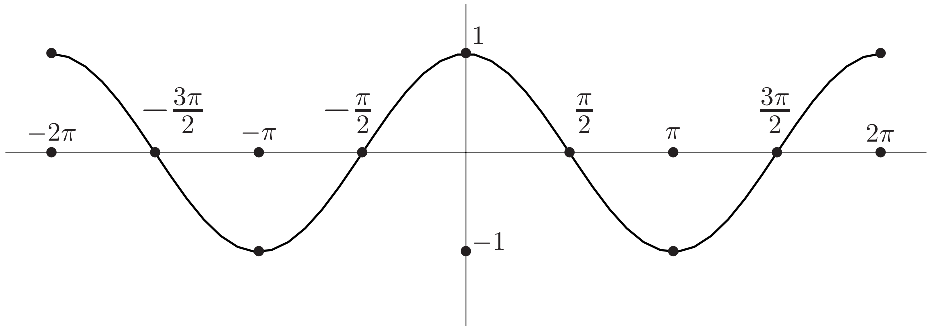
13.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	und	0

Note: "und" means undefined.

Return to Problem Review Topic 12

14.



Return to Problem Review Topic 13

15. a) $$\frac{1+\sec\theta}{\csc\theta} = \frac{1}{\csc\theta} + \frac{\sec\theta}{\csc\theta} = \sin\theta + \frac{\sin\theta}{\cos\theta} = \sin\theta + \tan\theta$$

Return to Problem Review Topic 14

15. b) $(1-\cos^2\theta)(1+\cot^2\theta) = (\sin^2\theta)(\csc^2\theta) = \sin^2\theta\left(\dfrac{1}{\sin^2\theta}\right) = 1$

Return to Problem Review Topic 14

15. c)

$$\begin{aligned}
\tan\theta + \frac{\cos\theta}{1+\sin\theta} &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{(1+\sin\theta)} \\
&= \frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)} + \frac{\cos\theta\cos\theta}{(1+\sin\theta)\cos\theta} \\
&= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{1}{\cos\theta} \\
&= \sec\theta
\end{aligned}$$

Return to Problem Review Topic 14

16. a) $\arcsin(1) = \dfrac{\pi}{2}$

Return to Problem Review Topic 15

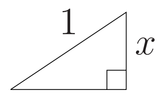
16. b) $\arccos(0) = \dfrac{\pi}{2}$

Return to Problem Review Topic 15

16. c) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

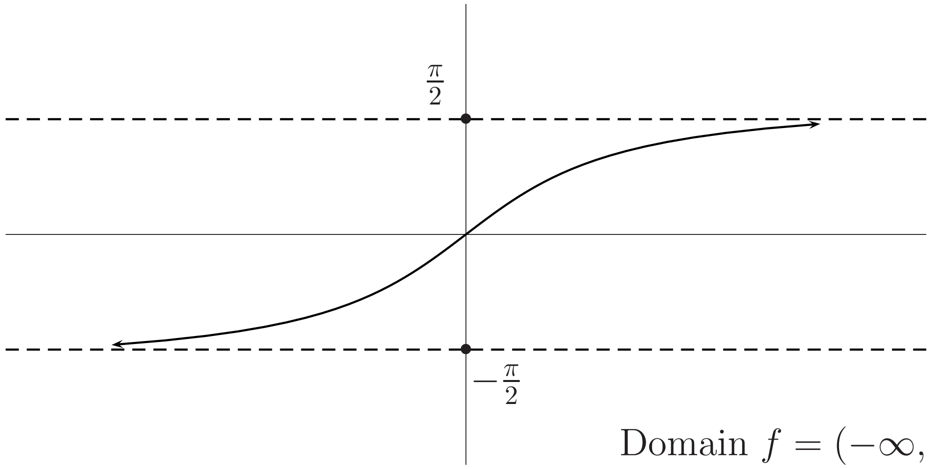
Return to Problem Review Topic 15

16. d) Arcsin(x) means the angle whose sine is $x = \dfrac{x}{1}$. The picture below has an angle whose sine is $\dfrac{x}{1}$. Using the Pythagorean Theorem, the missing side is $\sqrt{1-x^2}$. Thus, $\tan(\arcsin(x)) = \dfrac{x}{\sqrt{1-x^2}}$.



Return to Problem Review Topic 15

17.



Domain $f = (-\infty, \infty)$

Range $f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Return to Problem Review Topic 15

18. a) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ when θ is in the second or third quandrants. Thus, $\theta = \dfrac{5\pi}{6}$ or $\dfrac{7\pi}{6}$. Since $\cos x$ is 2π periodic, $\theta = \dfrac{5\pi}{6} + 2k\pi$ or $\dfrac{7\pi}{6} + 2k\pi$, for any integer k.

Return to Problem Review Topic 16

18. b) Factor the expression like a quadratic. That is,

$$2\sin^2\theta - \sin\theta - 1 = 0, \quad 0 \le \theta < 2\pi$$
$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$\begin{array}{r|l} 2\sin\theta + 1 = 0 & \sin\theta = 1 \\ \sin\theta = -\dfrac{1}{2} & \theta = \dfrac{\pi}{2} \\ \theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6} & \end{array}$$

Solution: $\theta = \dfrac{\pi}{2}, \dfrac{7\pi}{6}$, or $\dfrac{11\pi}{6}$

Return to Problem Review Topic 16

18. c) $\sin\theta + \sin 2\theta = 0$. Try to get all arguments in terms of θ. So,

$$\sin\theta + 2\sin\theta\cos\theta = 0, \quad 0 \le \theta < 2\pi$$
$$\sin\theta(1 + 2\cos\theta) = 0$$

$\sin\theta = 0$	$1 + 2\cos\theta = 0$
$\theta = 0,\ \pi$	$\cos\theta = -\frac{1}{2}$
	$\theta = \frac{2\pi}{3},\ \frac{4\pi}{3}$

Solution: $\theta = 0,\ \frac{2\pi}{3}, \pi,$ or $\frac{4\pi}{3}$

Return to Problem Review Topic 16

18. d) $\sin\theta - \cos\theta = 0, \quad 0 \le \theta < 2\pi$

$$\sin\theta = \cos\theta$$

Based on the unit circle definition of the trigonometric functions (Review Topic 11), the question really is asking when the x and y coordinates of a point on the unit circle are equal. This happens when $\theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$.

Solution: $\theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

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Math Fudge Brownies

(8 x 8 x 2 pan)

1/2	cup butter
2	squares (1 ounce each) Bakers unsweetened chocolate
1	cup sugar
2	eggs
1	teaspoon vanilla
3/4	cup flour
1/2	cup chopped walnuts

Grease an 8 x 8 x 2 - inch pan. Slowly melt butter and chocolate very carefully over low heat (hint: place the chocolate squares on top of the stick of butter so that the butter melts first). Remove from heat; stir in sugar. Add eggs and vanilla. Do not stir too much or brownies will rise too high, then fall and be dry! Stir in flour and nuts. Spread batter in pan. Bake in a 350 degree oven for 25 minutes. Do not over bake. If your oven heats from the bottom put the pan on an air-bake cookie sheet so that the bottom will not over cook. Cool. Cut into bars and wrap tightly.

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