

Name \_\_\_\_\_

Section \_\_\_\_\_

Instructor \_\_\_\_\_

**INSTRUCTIONS:** There 13 problems on 9 pages. Please check that your copy of the exam has all the pages and all the problems numbered 1 to 13.

Work in a neat and organized manner. All work must be shown on all problems. Full credit will not be given unless you have work to back up your answers, and your work is clearly shown. No credit will be given for numerical approximations to your answer without all the supporting work.

All work must be on these pages to be graded. If you use scratch paper, you **MUST** transfer your work to this test.

A scientific calculator will be permitted on the final exam; however, calculators with graphic, word-processing, symbolic manipulation, or programming capabilities will *not* be permitted for this exam (*no graphing, Pro, Plus, Multiview calculators etc. allowed*).

**NO CELL PHONES ALLOWED. \*Anyone caught with a cellphone will earn a grade of 0%.**

Problem	Points	Score
1	28	
2	12	
3	12	
4	8	
5	12	
6	22	
7	32	
8	8	
9	14	
10	12	
11	12	
12	14	
13	14	
Total	200	

(28) 1. Compute the derivative of the following functions. **Do NOT simplify.**

(a)  $f(x) = \frac{1-2x}{3x^2+x}$

(b)  $f(x) = \ln(3x^2 - 7x + 1)$

(c)  $g(x) = \frac{2}{\sqrt{x}} + \frac{5}{x} - 11$

(d)  $h(x) = (x + 6)e^{2x}$

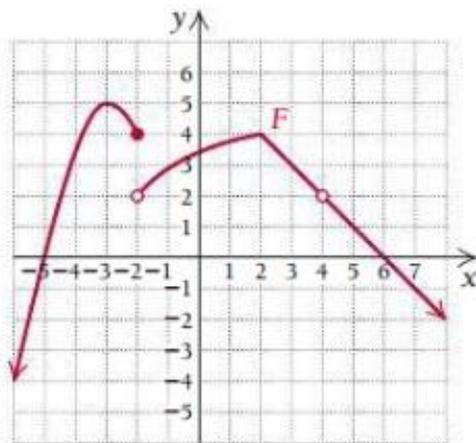
(12) 2. Calculate the following limits.

(a)  $\lim_{x \rightarrow -2} \frac{x^2+1}{3x^2+2x-1}$

(b)  $\lim_{x \rightarrow \infty} \frac{4x+2x^2-5x^3}{7x^3-2x^2-x+20}$

(c)  $\lim_{x \rightarrow 3} \frac{2x^2-18}{x^2+x-12}$

(12) 3. Suppose that the graph of  $y = F(x)$  is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a)  $\lim_{x \rightarrow -2^-} F(x) =$

(c)  $\lim_{x \rightarrow 2^+} F(x) =$

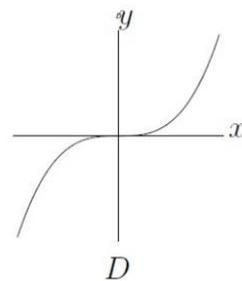
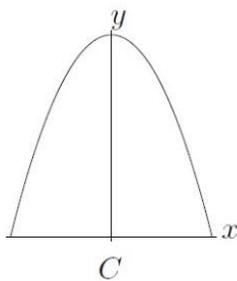
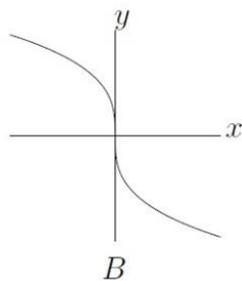
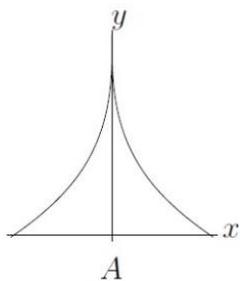
(b)  $\lim_{x \rightarrow -2} F(x) =$

(d)  $\lim_{x \rightarrow 4} F(x) =$

(8) 4. A company is planning to manufacture and market a new single pod coffee maker. After conducting extensive market surveys, the research department provides the following cost function estimate. Find the marginal cost (to the nearest cent) at a production level of 20 units for the cost function

$$C(x) = (0.02x^2 + 1.6)^{3/2} . \text{ Include units on your answer.}$$

(12) 5. Match each description below to one of the graphs of  $f(x)$ .



(a) \_\_\_\_\_  $f'(x) > 0$  and  $f''(x) < 0$  on  $(-\infty, 0)$ ,  $f'(x) < 0$  and  $f''(x) < 0$  on  $(0, \infty)$

(b) \_\_\_\_\_  $f'(x) > 0$  and  $f''(x) < 0$  on  $(-\infty, 0)$ ,  $f'(x) > 0$  and  $f''(x) > 0$  on  $(0, \infty)$

(c) \_\_\_\_\_  $f'(x) < 0$  and  $f''(x) < 0$  on  $(-\infty, 0)$ ,  $f'(x) < 0$  and  $f''(x) > 0$  on  $(0, \infty)$

(d) \_\_\_\_\_  $f'(x) > 0$  and  $f''(x) > 0$  on  $(-\infty, 0)$ ,  $f'(x) < 0$  and  $f''(x) > 0$  on  $(0, \infty)$

(22) 6. Let  $f(x) = x^3 - 6x^2 + 9x + 1$ .

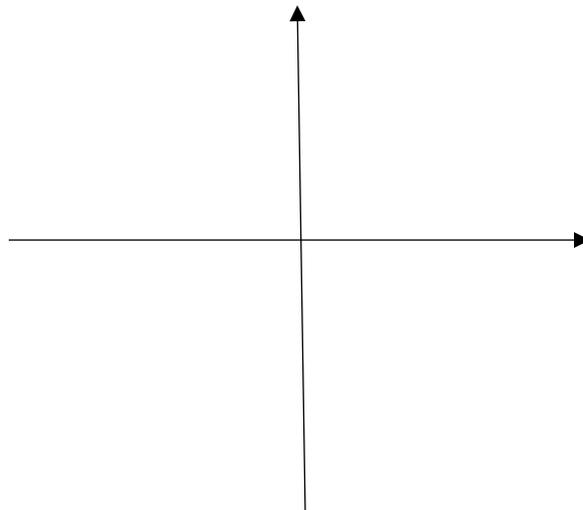
(a) Find the critical values of this function.

(b) Using calculus, give the interval(s) in interval notation over which  $f$  is increasing and/or decreasing.

(c) Using your answer from (b), find the relative extrema of  $f$ . State as ordered pairs.

(d) Using calculus, give the interval(s) in interval notation over which  $f$  is concave up and/or down.

(e) Sketch the graph of  $f(x)$  on the given axes. Mark the relative extrema and inflection point. Plot additional points as needed.



(32) 7. Compute the following integrals.

(a)  $\int \left( 5x^{3/2} + \frac{2}{x} - 3 \right) dx$

(b)  $\int \left( \sqrt{x} + \frac{2}{x^2} + 8x \right) dx$

(c)  $\int_0^1 x\sqrt{x^2 + 8} dx$

(d)  $\int xe^{-4x} dx$  (Use integration by parts.)

- (8) 8. Since 2013, the annual rate of change in the national consumer revolving credit market debt can be modeled by the function

$$D'(t) = 12.922t + 11.474$$

where  $D'(t)$  is in billions of dollars per year and  $t$  is the number of years since January 2013. Using the initial condition, find  $D(t)$  if in 2023 the debt was \$782 billion.

- (14) 9. Using the two curves  $f(x) = 2x + 5$  and  $g(x) = x^2 + 2x + 1$ ,

a) Using algebra, find the points of intersection as ordered pairs.

b) Sketch a quick graph and shade the region bounded by the two curves.

c) Set up, **but do not solve**, the integral that represents the area bounded by the two curves.

(12) 10. A manufacturer of fertilizer finds that the national sales of fertilizer roughly follow the seasonal pattern

$$f(x, y) = -x^3 + 4xy - 2y^2 + 1$$

where  $f$  is measured in thousands of pounds. **The critical points of  $f(x, y)$  are  $(0, 0)$  and  $(\frac{4}{3}, \frac{4}{3})$ .** Identify each critical point as a relative minimum, a relative maximum, or a saddle point.

(12) 11. Airbnb has found that the demand for tiny houses in a certain U.S. region is represented by  $D(x) = 200 - 4x - x^2$ , and the supply by  $S(x) = x^2 - 4x + 38$ , where  $x$  is the number of rentals, in hundreds, and  $D(x)$  and  $S(x)$  are the rates in dollars per rental booking, per month.

(a) Find the equilibrium point. State your answer as an ordered pair.

(b) Find the consumer surplus at the equilibrium point. Include units on your answer.

(14)12. Let  $f(a, b) = a^3b - 2be^{-a} + 5a^2b^2$ . Find:

(a)  $f_a$

(b)  $f_{ab}$

(c)  $f_b(2, 1)$

(14) 13. A local lumber yard stocks large pallets of two types of lumber: pressure-treated and untreated. Use the method of Lagrange multipliers to find the minimum value of the company's cost function (in thousands of dollars)

$$C(x, y) = x^2 + 3y^2 + 10$$

subject to the supply chain constraint of  $g(x, y) = 8 - x - y$ , where  $x$  is the number of pressure-treated pallets and  $y$  is the number of untreated pallets. State your answer in sentence form with correct units on your answer.